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DECLINATION, RADIAL DISTANCE, AND PHASES OF THE MOON FOR THE YEARS 1961 TO 1971 FOR USE IN TRAJECTORY CONSIDERATIONS

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

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SUMMARY

As a byproduct of the preparation of solar and lunar coordinates for use in trajectory calculations a time history has been obtained of the radial distance and declination of the moon and its phases. Results are intended for use as an aid in the selection of launch dates. Results are presented for the years 1961 to 1971 in a form which permits a rapid approximate determination of the combination of declination and lighting for any calendar date. The information provides a time basis for entering tables of the moon's coordinates to obtain more precise data for use in computing insertion conditions.

INTRODUCTION

Extensive analytical studies of lunar and circumlunar space vehicle trajectories are currently being performed with much emphasis being placed on the design of trajectories for the period between 1964 and 1968. In performing such studies at the Langley Research Center, use is being made of a four-body, three-dimensional lunar trajectory program which operates in terms of actual calendar dates and uses positions of the bodies as obtained from U.S. Naval Observatory data. The particular program in use has been obtained by adapting for efficient application to lunar missions an interplanetary trajectory program described in reference 1.

In designing a nominal trajectory some of the many factors which influence the selection of firing dates are the declination of the moon, its distance from the earth, and its lighting. Launch times are being selected for circumlunar missions, for example, which place a vehicle in the vicinity of the moon when the moon is at a negative declination and when portions of the lunar surface of interest in the mission will be suitably illuminated.

Information on the position and phases of the moon is usually not available for more than 2 years in advance, but as a byproduct of the preparation of the Naval Observatory data for use in trajectory calculations, it has been possible to perform some additional manipulations which provide an approximate calendar of the phases and declination of the moon and its radial distance from the earth for the years 1961 to 1971. The material was originally developed for use with the particular lunar trajectory computing program at hand but should be of general interest to others involved in similar studies. While the results presented herein do not provide sufficient data on the moon's position for the complete determination of injection conditions, they will provide a rapid visual determination of a time basis for interpolating in an ephemeris of the moon to obtain such data.

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A discussion is given of the form and nature of the Naval Observatory data employed and of the transformations and approximations which have been used. Results include tables of the dates for new moon, first quarter, full moon, and last quarter and plots of the moon's declination and distance from the earth as a function of calendar date. Although precise results for the years 1961 and 1962 are available in references 2 and 3, data for these years, obtained by the procedures used herein, are included in order to provide some basis for the assessment of the accuracy of the present methods.

SYMBOLS

d,h,m day, hour, and minute, respectively

N_{JD} Julian day number

r_M distance from earth to moon, measured in earth radii (based on an equatorial earth radius of 3,963.208 statute miles)

T time in Julian centuries of 36,525 days, equation (3)

X,Y,Z equatorial rectangular coordinates

$X_x,Y_x,Z_x, \ldots$ elements of rotation matrix defined by equations (1) and (2)

$\rho = \sqrt{X^2 + Y^2 + Z^2}$

α right ascension, angular arc measured eastward from γ in equatorial plane

δ declination, angular distance measured north or south from equatorial plane

λ longitude, angular arc measured eastward from γ along ecliptic

$\Delta\lambda = \lambda_M - \lambda_S$

β latitude, angular distance measured in a plane perpendicular to ecliptic

γ vernal equinox, the point of intersection of celestial equator with ecliptic through which sun passes equator from south to north

ϵ obliquity of ecliptic, angle between plane of ecliptic and plane of equator

π_M horizontal parallax, angle subtended by equatorial radius of earth as viewed from moon

Subscripts:

M moon

S sun

o value at initial or reference time

Abbreviations:

FQ first quarter

LQ last quarter

DATA EMPLOYED

In expressing the coordinates of the various bodies two fundamental reference planes are used. These are the plane of the earth's equator and the plane of the ecliptic at certain specified times. The line of intersection of the two planes defines the vernal equinox γ and provides the fundamental direction.

Two equatorial coordinate systems will be used. One provides the geocentric rectangular coordinates of a body while the other provides

the equatorial polar coordinates: right ascension α, measured eastward from γ along the equator, and declination δ, measured north or south from the equator. In addition, an ecliptic polar coordinate system will be used in which a body's position is defined by its longitude λ, measured eastward from γ along the ecliptic, and latitude β, measured from the ecliptic. Relationships between the various coordinate systems are indicated in figure 1 and are discussed in the appendix.

The U.S. Naval Observatory data employed in the calculations consists of punched cards containing the geocentric equatorial rectangular coordinates of the sun referred to the mean equinox of 1950.0 and the apparent longitude of the moon referred to the mean equinox of date, the apparent latitude of the moon referred to the ecliptic of date, and the moon's horizontal parallax. Data in these forms are available for the sun for the years 1800 to 2000 and for the moon for the years 1960 to 1971. Solar positions are given at 4-day intervals and lunar positions are given at 1/2-day intervals. Fifth-order interpolation has been used in the present calculations for intermediate dates.

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APPROXIMATIONS IN THE CALCULATIONS

The precise determination of the times of new moon, first quarter, full moon, and last quarter as performed by the Nautical Almanac Office of the U.S. Naval Observatory consists of determining "the times at which the excess of the apparent longitude of the moon over the apparent longitude of the sun is 0°, 90°, 180°, and 270°, respectively." In the present calculations the determination of the phases has been approximated by comparing the geometric position of the sun with the apparent position of the moon. While reduction of the sun's geometric longitude to apparent longitude should involve consideration of aberration and of the precession and nutation in longitude, only the precession has been considered. For the period of interest the combined contributions of nutation and aberration have been examined and it has been found that they should not exceed approximately 30 seconds of arc, corresponding in the present study to a discrepancy in time of the moon's phases of about 1 minute.

A related approximation involves the use of a mean obliquity of the ecliptic rather than true obliquity in transforming from the sun's geocentric equatorial rectangular coordinates to the longitude. Specifically, in the calculations of the longitude throughout any given year, the value of the mean obliquity at the beginning of that year as tabulated in reference 4 has been used. This same approximation is involved in the computations of the declination of the moon. For present purposes its effect should be negligible.

Where comparison with Naval Observatory calculations is possible (that is, for 1961 and 1962) the results of the present determination of the moon's phases do not differ by more than 1 minute of time. There is some indication of an increasing tendency of the results to depart from those of the Naval Observatory as time progresses. The intended use of the present results as simply a guide to a date for entering an accurate ephemeris would not be seriously affected, however, by even a considerable departure from precise results.

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DETERMINATION OF THE RADIAL DISTANCE, DECLINATION, AND PHASES OF THE MOON

In this section the procedures employed in the calculations are briefly described. The calculations represent the appropriate application of conventional transformations between the various coordinate systems in use; for ease of reference these transformations are recorded in the appendix to this paper.

The radial distance r_M of the moon has been obtained directly from the values of horizontal parallax π_M provided by the Naval Observatory data. It is given in terms of equatorial earth radii by the expression

$$r_M = \frac{1}{\sin \pi_M}$$

The declination δ_M of the moon as a function of calendar date has been obtained by a transformation from the values of the longitude λ_M and latitude β_M contained in the Naval Observatory data and from the mean obliquity ϵ of the ecliptic tabulated in reference 4. The appropriate transformation is obtained from equations (10) of the appendix as

$$\delta_M = \sin^{-1}\left(\sin \beta_M \cos \epsilon + \cos \beta_M \sin \epsilon \sin \lambda_M\right)$$

where $-\frac{\pi}{2} < \delta_M < \frac{\pi}{2}$.

The times of new moon, first quarter, full moon, and last quarter have been obtained as the times at which the apparent longitude of the moon exceeds the geometric longitude of the sun by 0°, 90°, 180°, and 270°, respectively. As noted previously, a precise determination would involve the use of apparent longitudes for both the sun and the moon.

The longitude of the moon referred to the equinox of date is obtainable directly by interpolation in the tabulated values provided by the Naval Observatory. In order to obtain the longitude of the sun, the equatorial rectangular coordinates provided by the Naval Observatory data must first be rotated in the equatorial plane from the equinox of 1950.0 to the equinox of date through use of equations (1), (2), and (3) and then transformed to longitude and latitude in the ecliptic system by use of equations (12) and (13).

An iterative procedure based on a relationship between the difference in longitude of the two bodies and time has been developed which converges on the dates for the moon's phases. The calculation is based on determining initially, for example, the time for a new moon and is begun by selecting a date near the date for a known new moon. For this date the longitudes λ_S and λ_M of the sun and the moon, respectively, can be obtained from the Naval Observatory data in the manner described. The difference between the two longitudes, designated $\Delta\lambda = \lambda_M - \lambda_S$, is divided by 12.2°, the average daily eastward travel of the moon relative to the sun, to determine an appropriate increment to the initial time estimate.

The new time estimate is used with the tabulated Naval Observatory data to obtain a second value of $\Delta\lambda$. The process is repeated until, for a new moon, $\Delta\lambda$ is reduced to some value acceptably close to zero.

When the time of the new moon has been established, the time for first quarter is sought. For this purpose the date corresponding to the new moon is increased by 7.375 days (approximately a quarter of the average synodic month), the condition for convergence is changed to $\Delta\lambda = \lambda_M - \left(\lambda_S + 90^{\circ}\right)$, and the calculation proceeds until, again, $\Delta\lambda$ is reduced to an acceptable level.

In a similar manner the times for full moon and last quarter are found by successively setting the condition for convergence to $\Delta\lambda = \lambda_M - \left(\lambda_S + 180^{\circ}\right)$ and $\Delta\lambda = \lambda_M - \left(\lambda_S + 270^{\circ}\right)$.

PRESENTATION OF RESULTS

Results of the calculations are given in tabular and graphical form. Table I serves simply to identify a particular month and date with the corresponding number of the day in the year after January 0. It is intended to be used in conjunction with the plots of the declination and distance of the moon. Two forms are given: one for nonleap years, the other for leap years.

Tables II(a) to II(k) contain the calculated (universal) times of the phases of the moon for the years 1961 to 1971, respectively. Figures 2 to 12 present plots for 1961 to 1971, respectively, of the radial distance r_M and declination of the moon δ_M as a function of the day of the year, numbered from January 0. Included on the plots of the declination are symbols indicating the principal phases of the moon. These provide a rapid visual determination of the combination of lunar lighting and declination for any date.

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Langley Research Center,
National Aeronautics and Space Administration,
Langley Field, Va., May 15, 1961.

APPENDIX

COORDINATE TRANSFORMATIONS

As stated in the text, the calculations reported herein represent the appropriate application of conventional transformations between the various coordinate systems commonly in use in astronomy. For ease of reference they are assembled and described in this appendix.

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Reduction of Rectangular Coordinates to the Equinox of Date

The sun's position is provided by the Naval Observatory in the form of geocentric equatorial rectangular coordinates referred to the equinox of 1950.0. A first step in determining the sun's longitude is the reduction of these coordinates to the equinox of date. This is readily accomplished through the use of the following formula, described in reference 4:

$$\begin{Bmatrix} X \\ Y \\ Z \end{Bmatrix} = \begin{bmatrix} X_x & Y_x & Z_x \\ X_y & Y_y & Z_y \\ X_z & Y_z & Z_z \end{bmatrix} \begin{Bmatrix} X_o \\ Y_o \\ Z_o \end{Bmatrix} \tag{1}$$

where, in the present case, X_o, Y_o, and Z_o are the equatorial rectangular coordinates referred to the equinox of 1950.0, and X, Y, and Z are the equatorial rectangular coordinates referred to the equinox of date. The elements X_x, X_y, . . ., Z_z of the rotation matrix for the transformation from 1950.0 are given by reference 4 as:

$$\left.\begin{array}{rl}
X_x = & 1.00000000 - 0.00029697\ T^2 - 0.00000013\ T^3 \\
Y_x = -X_y = & -0.02234988\ T - 0.00000676\ T^2 + 0.00000221\ T^3 \\
Z_x = -X_z = & -0.00971711\ T + 0.00000207\ T^2 + 0.00000096\ T^3 \\
Y_y = & 1.00000000 - 0.00024976\ T^2 - 0.00000015\ T^3 \\
Y_z = Z_y = & -0.00010859\ T^2 - 0.00000003\ T^3 \\
Z_z = & 1.00000000 - 0.00004721\ T^2 + 0.00000002\ T^3
\end{array}\right\} \tag{2}$$

where T is the number of Julian centuries of 36,525 days since 1950.0. A suitable expression for T is

$$T = \frac{N_{JD,date} - N_{JD,o}}{36,525} \tag{3}$$

where $N_{JD,date}$ denotes the Julian day number of the date for which the coordinate reduction is to be performed and $N_{JD,o} = 2433282.5$.

Relationships Between Polar and Rectangular Coordinates

Expressions relating the equatorial rectangular coordinates of a body to its polar coordinates in either the equatorial or the ecliptic systems are contained, for example, in reference 5. A review of their developments is included here.

In arriving at the desired expressions the following relations between the sides a, b, and c and angles A, B, and C of a spherical triangle will be employed:

$$\cos a = \cos b \cos c + \sin b \sin c \cos A \tag{4}$$

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \tag{5}$$

$$\sin a \cos B = \cos b \sin c - \sin b \cos c \cos A \tag{6}$$

The relationships between the equatorial rectangular coordinates of a body and its right ascension α, declination δ, longitude λ, and latitude β can be obtained by consideration of figure 1.

In the left-hand diagram in figure 1 the angular coordinates correspond to the following arcs:

$$\alpha = \gamma D$$

$$\delta = SD$$

$$\lambda = \gamma A$$

$$\beta = SA$$

and the following definitions apply:

$$\left.\begin{aligned} \rho &= \sqrt{X^2 + Y^2 + Z^2} \\ X &= \rho \cos S\gamma \\ Y &= \rho \cos SR \\ Z &= \rho \cos SP \end{aligned}\right\} \tag{7}$$

From equation (4) and the center diagram in figure 1 may be obtained

$$\cos S\gamma = \cos \alpha \cos \delta$$

$$\cos SR = \cos(90^\circ - \alpha)\cos \delta = \sin \alpha \cos \delta$$

$$\cos SP = \cos(90^\circ - \delta) = \sin \delta$$

so that equations (7) yield

$$\left.\begin{aligned} X &= \rho \cos \alpha \cos \delta \\ Y &= \rho \sin \alpha \cos \delta \\ Z &= \rho \sin \delta \end{aligned}\right\} \tag{8}$$

Equations (8) may be used to express the equatorial polar coordinates, right ascension and declination, as the following functions of the rectangular coordinates:

$$\left.\begin{aligned} \alpha &= \tan^{-1} \frac{Y}{X} \\ \delta &= \sin^{-1} \frac{Z}{\rho} \end{aligned}\right\} \tag{9}$$

To relate the ecliptic polar coordinates, longitude λ and latitude β, of a body to its equatorial rectangular coordinates use is made of equations (4), (5), and (6) in application to the right-hand diagram in figure 1 to obtain the relationships

$$\left.\begin{aligned} \sin \delta &= \sin \beta \cos \epsilon + \cos \beta \sin \epsilon \sin \lambda \\ \cos \alpha \cos \delta &= \cos \beta \cos \lambda \\ -\cos \delta \sin \alpha &= \sin \beta \sin \epsilon - \cos \beta \cos \epsilon \sin \lambda \end{aligned}\right\} \tag{10}$$

Combining equations (8) and (10) gives

$$\left.\begin{aligned} X &= \rho \cos\beta \cos\lambda \\ Y &= \rho(\cos\beta \cos\epsilon \sin\lambda - \sin\beta \sin\epsilon) \\ Z &= \rho(\cos\beta \sin\epsilon \sin\lambda + \sin\beta \cos\epsilon) \end{aligned}\right\} \tag{11}$$

Since $-\frac{\pi}{2} \leqq \beta \leqq \frac{\pi}{2}$, $\cos\beta$ is always positive so that the first of equations (11) may be used to obtain the magnitude of λ as

$$\lambda = \cos^{-1} \frac{|X|}{\rho \cos\beta} \tag{12}$$

The appropriate quadrant may be assigned by noting that λ lies in the same quadrant as $\tan^{-1} \frac{Y}{X}$. An expression for β for use in equation (12) may be obtained by noting that

$$Z \cos\epsilon - Y \sin\epsilon = \rho \sin\beta$$

so that

$$\beta = \sin^{-1} \frac{Z \cos\epsilon - Y \sin\epsilon}{\rho} \tag{13}$$

Equations (12) and (13) are used with equations (1), (2), and (3) in computing the sun's longitude for use in the determination of the moon's phases. The first of equations (10) is used to obtain the moon's declination.

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TABLE I.- CALENDAR SHOWING NUMBER OF DAY IN YEAR

AFTER JANUARY 0

(a) Nonleap years: 1961, 1962, 1963, 1965, 1966, 1967, 1969, 1970, and 1971

Day of month	Day of year											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	60	91	121	152	182	213	244	274	305	335
2	2	33	61	92	122	153	183	214	245	275	306	336
3	3	34	62	93	123	154	184	215	246	276	307	337
4	4	35	63	94	124	155	185	216	247	277	308	338
5	5	36	64	95	125	156	186	217	248	278	309	339
6	6	37	65	96	126	157	187	218	249	279	310	340
7	7	38	66	97	127	158	188	219	250	280	311	341
8	8	39	67	98	128	159	189	220	251	281	312	342
9	9	40	68	99	129	160	190	221	252	282	313	343
10	10	41	69	100	130	161	191	222	253	283	314	344
11	11	42	70	101	131	162	192	223	254	284	315	345
12	12	43	71	102	132	163	193	224	255	285	316	346
13	13	44	72	103	133	164	194	225	256	286	317	347
14	14	45	73	104	134	165	195	226	257	287	318	348
15	15	46	74	105	135	166	196	227	258	288	319	349
16	16	47	75	106	136	167	197	228	259	289	320	350
17	17	48	76	107	137	168	198	229	260	290	321	351
18	18	49	77	108	138	169	199	230	261	291	322	352
19	19	50	78	109	139	170	200	231	262	292	323	353
20	20	51	79	110	140	171	201	232	263	293	324	354
21	21	52	80	111	141	172	202	233	264	294	325	355
22	22	53	81	112	142	173	203	234	265	295	326	356
23	23	54	82	113	143	174	204	235	266	296	327	357
24	24	55	83	114	144	175	205	236	267	297	328	358
25	25	56	84	115	145	176	206	237	268	298	329	359
26	26	57	85	116	146	177	207	238	269	299	330	360
27	27	58	86	117	147	178	208	239	270	300	331	361
28	28	59	87	118	148	179	209	240	271	301	332	362
29	29		88	119	149	180	210	241	272	302	333	363
30	30		89	120	150	181	211	242	273	303	334	364
31	31		90		151		212	243		304		365

TABLE I.- CALENDAR SHOWING NUMBER OF DAY IN YEAR

AFTER JANUARY 0 - Concluded

(b) Leap years: 1964 and 1968

Day of month	Day of year											
	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
1	1	32	61	92	122	153	183	214	245	275	306	336
2	2	33	62	93	123	154	184	215	246	276	307	337
3	3	34	63	94	124	155	185	216	247	277	308	338
4	4	35	64	95	125	156	186	217	248	278	309	339
5	5	36	65	96	126	157	187	218	249	279	310	340
6	6	37	66	97	127	158	188	219	250	280	311	341
7	7	38	67	98	128	159	189	220	251	281	312	342
8	8	39	68	99	129	160	190	221	252	282	313	343
9	9	40	69	100	130	161	191	222	253	283	314	344
10	10	41	70	101	131	162	192	223	254	284	315	345
11	11	42	71	102	132	163	193	224	255	285	316	346
12	12	43	72	103	133	164	194	225	256	286	317	347
13	13	44	73	104	134	165	195	226	257	287	318	348
14	14	45	74	105	135	166	196	227	258	288	319	349
15	15	46	75	106	136	167	197	228	259	289	320	350
16	16	47	76	107	137	168	198	229	260	290	321	351
17	17	48	77	108	138	169	199	230	261	291	322	352
18	18	49	78	109	139	170	200	231	262	292	323	353
19	19	50	79	110	140	171	201	232	263	293	324	354
20	20	51	80	111	141	172	202	233	264	294	325	355
21	21	52	81	112	142	173	203	234	265	295	326	356
22	22	53	82	113	143	174	204	235	266	296	327	357
23	23	54	83	114	144	175	205	236	267	297	328	358
24	24	55	84	115	145	176	206	237	268	298	329	359
25	25	56	85	116	146	177	207	238	269	299	330	360
26	26	57	86	117	147	178	208	239	270	300	331	361
27	27	58	87	118	148	179	209	240	271	301	332	362
28	28	59	88	119	149	180	210	241	272	302	333	363
29	29	60	89	120	150	181	211	242	273	303	334	364
30	30		90	121	151	182	212	243	274	304	335	365
31	31		91		152		213	244		305		366

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TABLE II.- PHASES OF THE MOON

[Universal time]

(a) 1961

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
								Jan.	1	23	7	Jan.	10	3	3
Jan.	16	21	31	Jan.	23	16	14	Jan.	31	18	48	Feb.	8	16	50
Feb.	15	8	11	Feb.	22	8	35	Mar.	2	13	36	Mar.	10	2	58
Mar.	16	18	51	Mar.	24	2	49	Apr.	1	5	48	Apr.	8	10	17
Apr.	15	5	38	Apr.	22	21	50	Apr.	30	18	41	May	7	15	58
May	14	16	55	May	22	16	19	May	30	4	38	June	5	21	19
June	13	5	17	June	21	9	2	June	28	12	38	July	5	3	33
July	12	19	12	July	20	23	14	July	27	19	51	Aug.	3	11	48
Aug.	11	10	37	Aug.	19	10	52	Aug.	26	3	14	Sept.	1	23	6
Sept.	10	2	51	Sept.	17	20	24	Sept.	24	11	34	Oct.	1	14	11
Oct.	9	18	54	Oct.	17	4	35	Oct.	23	21	31	Oct.	31	9	0
Nov.	8	9	59	Nov.	15	12	13	Nov.	22	9	45	Nov.	30	6	20
Dec.	7	23	53	Dec.	14	20	6	Dec.	22	0	43	Dec.	30	3	58

TABLE II.- PHASES OF THE MOON - Continued

[Universal time]

(b) 1962

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
Jan.	6	12	36	Jan.	13	5	2	Jan.	20	18	17	Jan.	28	23	37
Feb.	5	0	11	Feb.	11	15	44	Feb.	19	13	19	Feb.	27	15	51
Mar.	6	10	32	Mar.	13	4	40	Mar.	21	7	57	Mar.	29	4	12
Apr.	4	19	46	Apr.	11	19	51	Apr.	20	0	34	Apr.	27	13	0
May	4	4	26	May	11	12	45	May	19	14	33	May	26	19	6
June	2	13	28	June	10	6	22	June	18	2	3	June	24	23	43
July	1	23	53	July	9	23	40	July	17	11	41	July	24	4	19
July	31	12	25	Aug.	8	15	56	Aug.	15	20	10	Aug.	22	10	27
Aug.	30	3	10	Sept.	7	6	46	Sept.	14	4	12	Sept.	20	19	37
Sept.	28	19	41	Oct.	6	19	55	Oct.	13	12	34	Oct.	20	8	48
Oct.	28	13	6	Nov.	5	7	16	Nov.	11	22	4	Nov.	19	2	10
Nov.	27	6	31	Dec.	4	16	49	Dec.	11	9	28	Dec.	18	22	43
Dec.	26	23	0												

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TABLE II.- PHASES OF THE MOON - Continued

[Universal time]

(c) 1963

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
				Jan.	3	1	3	Jan.	9	23	9	Jan.	17	20	35
Jan.	25	13	43	Feb.	1	8	51	Feb.	8	14	53	Feb.	16	17	40
Feb.	24	2	7	Mar.	2	17	18	Mar.	10	7	50	Mar.	18	12	9
Mar.	25	12	11	Apr.	1	3	16	Apr.	9	0	58	Apr.	17	2	54
Apr.	23	20	30	Apr.	30	15	9	May	8	17	25	May	16	13	37
May	23	4	1	May	30	4	57	June	7	8	32	June	14	20	54
June	21	11	47	June	28	20	25	July	6	21	57	July	14	1	58
July	20	20	44	July	28	13	14	Aug.	5	9	32	Aug.	12	6	22
Aug.	19	7	36	Aug.	27	6	55	Sept.	3	19	35	Sept.	10	11	43
Sept.	17	20	52	Sept.	26	0	40	Oct.	3	4	45	Oct.	9	19	28
Oct.	17	12	44	Oct.	25	17	21	Nov.	1	13	56	Nov.	8	6	38
Nov.	16	6	52	Nov.	24	7	57	Nov.	30	23	55	Dec.	7	21	35
Dec.	16	2	8	Dec.	23	19	56	Dec.	30	11	5				

TABLE II.- PHASES OF THE MOON - Continued

[Universal time]

(d) 1964

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
												Jan.	6	15	59
Jan.	14	20	45	Jan.	22	5	30	Jan.	28	23	24	Feb.	5	12	43
Feb.	13	13	2	Feb.	20	13	25	Feb.	27	12	40	Mar.	6	10	1
Mar.	14	2	15	Mar.	20	20	40	Mar.	28	2	50	Apr.	5	5	46
Apr.	12	12	39	Apr.	19	4	10	Apr.	26	17	51	May	4	22	21
May	11	21	3	May	18	12	43	May	26	9	30	June	3	11	9
June	10	4	23	June	16	23	3	June	25	1	9	July	2	20	32
July	9	11	32	July	16	11	48	July	24	15	59	Aug.	1	3	30
Aug.	7	19	18	Aug.	15	3	21	Aug.	23	5	26	Aug.	30	9	16
Sept.	6	4	35	Sept.	13	21	25	Sept.	21	17	32	Sept.	28	15	3
Oct.	5	16	21	Oct.	13	16	58	Oct.	21	4	46	Oct.	27	22	0
Nov.	4	7	18	Nov.	12	12	21	Nov.	19	15	44	Nov.	26	7	12
Dec.	4	1	20	Dec.	12	6	3	Dec.	19	2	42	Dec.	25	19	28

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TABLE II.- PHASES OF THE MOON - Continued

[Universal time]

(e) 1965

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New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
Jan.	2	21	8	Jan.	10	21	1	Jan.	17	13	38	Jan.	24	11	8
Feb.	1	16	37	Feb.	9	8	54	Feb.	16	0	28	Feb.	23	5	40
Mar.	3	9	57	Mar.	10	17	53	Mar.	17	11	25	Mar.	25	1	38
Apr.	2	0	22	Apr.	9	0	41	Apr.	15	23	3	Apr.	23	21	8
May	1	11	57	May	8	6	21	May	15	11	53	May	23	14	41
May	30	21	14	June	6	12	12	June	14	2	1	June	22	5	37
June	29	4	53	July	5	19	37	July	13	17	3	July	21	17	54
July	28	11	46	Aug.	4	5	48	Aug.	12	8	23	Aug.	20	3	51
Aug.	26	18	51	Sept.	2	19	29	Sept.	10	23	33	Sept.	18	11	59
Sept.	25	3	19	Oct.	2	12	39	Oct.	10	14	15	Oct.	17	19	1
Oct.	24	14	12	Nov.	1	8	27	Nov.	9	4	16	Nov.	16	1	55
Nov.	23	4	11	Dec.	1	5	26	Dec.	8	17	22	Dec.	15	9	53
Dec.	22	21	4	Dec.	31	1	47								

TABLE II.- PHASES OF THE MOON - Continued

[Universal time]

(f) 1966

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
								Jan.	7	5	17	Jan.	13	20	1
Jan.	21	15	47	Jan.	29	19	49	Feb.	5	15	59	Feb.	12	8	54
Feb.	20	10	50	Feb.	28	10	16	Mar.	7	1	46	Mar.	14	0	20
Mar.	22	4	47	Mar.	29	20	44	Apr.	5	11	14	Apr.	12	17	29
Apr.	20	20	36	Apr.	28	3	50	May	4	21	2	May	12	11	20
May	20	9	43	May	27	8	51	June	3	7	41	June	11	4	59
June	18	20	10	June	25	13	24	July	2	19	37	July	10	21	44
July	18	4	31	July	24	19	1	Aug.	1	9	6	Aug.	9	12	57
Aug.	16	11	49	Aug.	23	3	3	Aug.	31	0	15	Sept.	8	2	8
Sept.	14	19	14	Sept.	21	14	26	Sept.	29	16	48	Oct.	7	13	9
Oct.	14	3	53	Oct.	21	5	36	Oct.	29	10	1	Nov.	5	22	19
Nov.	12	14	27	Nov.	20	0	21	Nov.	28	2	41	Dec.	5	6	23
Dec.	12	3	14	Dec.	19	21	42	Dec.	27	17	44				

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TABLE II.- PHASES OF THE MOON - Continued

[Universal time]

(g) 1967

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New moon	d	h	m	First quarter	d	h	m	Full moon	d	h	m	Last quarter	d	h	m
												Jan.	3	14	20
Jan.	10	18	7	Jan.	18	19	42	Jan.	26	6	41	Feb.	1	23	4
Feb.	9	10	45	Feb.	17	15	57	Feb.	24	17	44	Mar.	3	9	11
Mar.	11	4	31	Mar.	19	8	32	Mar.	26	3	22	Apr.	1	20	59
Apr.	9	22	21	Apr.	17	20	49	Apr.	24	12	4	May	1	10	34
May	9	14	56	May	17	5	19	May	23	20	23	May	31	1	53
June	8	5	15	June	15	11	13	June	22	4	58	June	29	18	40
July	7	17	1	July	14	15	54	July	21	14	40	July	29	12	15
Aug.	6	2	49	Aug.	12	20	45	Aug.	20	2	28	Aug.	28	5	36
Sept.	4	11	38	Sept.	11	3	6	Sept.	18	17	0	Sept.	26	21	45
Oct.	3	20	25	Oct.	10	12	12	Oct.	18	10	12	Oct.	26	12	5
Nov.	2	5	49	Nov.	9	1	1	Nov.	17	4	54	Nov.	25	0	24
Dec.	1	16	11	Dec.	8	17	58	Dec.	16	23	22	Dec.	24	10	49
Dec.	31	3	39												

TABLE II.- PHASES OF THE MOON - Continued

[Universal time]

(h) 1968

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
				Jan.	7	14	24	Jan.	15	16	12	Jan.	22	19	39
Jan.	29	16	30	Feb.	6	12	21	Feb.	14	6	44	Feb.	21	3	29
Feb.	28	6	56	Mar.	7	9	21	Mar.	14	18	53	Mar.	21	11	8
Mar.	28	22	49	Apr.	6	3	28	Apr.	13	4	52	Apr.	19	19	36
Apr.	27	15	22	May	5	17	55	May	12	13	6	May	19	5	45
May	27	7	31	June	4	4	47	June	10	20	14	June	17	18	14
June	25	22	25	July	3	12	42	July	10	3	18	July	17	9	12
July	25	11	50	Aug.	1	18	35	Aug.	8	11	33	Aug.	16	2	14
Aug.	23	23	57	Aug.	30	23	35	Sept.	6	22	8	Sept.	14	20	32
Sept.	22	11	9	Sept.	29	5	7	Oct.	6	11	47	Oct.	14	15	6
Oct.	21	21	45	Oct.	28	12	40	Nov.	5	4	26	Nov.	13	8	54
Nov.	20	8	2	Nov.	26	23	31	Dec.	4	23	8	Dec.	13	0	50
Dec.	19	18	19	Dec.	26	14	15								

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TABLE II.- PHASES OF THE MOON - Continued

[Universal time]

(i) 1969

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
								Jan.	3	18	28	Jan.	11	14	1
Jan.	18	4	59	Jan.	25	8	24	Feb.	2	12	57	Feb.	10	0	9
Feb.	16	16	26	Feb.	24	4	31	Mar.	4	5	18	Mar.	11	7	45
Mar.	18	4	52	Mar.	26	0	49	Apr.	2	18	46	Apr.	9	13	59
Apr.	16	18	16	Apr.	24	19	45	May	2	5	14	May	8	20	12
May	16	8	27	May	24	12	16	May	31	13	19	June	7	3	40
June	14	23	9	June	23	1	45	June	29	20	4	July	6	13	18
July	14	14	12	July	22	12	10	July	29	2	46	Aug.	5	1	39
Aug.	13	5	17	Aug.	20	20	4	Aug.	27	10	33	Sept.	3	16	58
Sept.	11	19	56	Sept.	19	2	25	Sept.	25	20	21	Oct.	3	11	6
Oct.	11	9	40	Oct.	18	8	32	Oct.	25	8	45	Nov.	2	7	14
Nov.	9	22	12	Nov.	16	15	46	Nov.	23	23	54	Dec.	2	3	51
Dec.	9	9	42	Dec.	16	1	10	Dec.	23	17	36	Dec.	31	22	53

TABLE II.- PHASES OF THE MOON - Continued

[Universal time]

(j) 1970

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
Jan.	7	20	36	Jan.	14	13	19	Jan.	22	12	56	Jan.	30	14	39
Feb.	6	7	13	Feb.	13	4	11	Feb.	21	8	19	Mar.	1	2	34
Mar.	7	17	43	Mar.	14	21	16	Mar.	23	1	53	Mar.	30	11	5
Apr.	6	4	10	Apr.	13	15	44	Apr.	21	16	22	Apr.	28	17	19
May	5	14	51	May	13	10	27	May	21	3	38	May	27	22	32
June	4	2	22	June	12	4	7	June	19	12	28	June	26	4	2
July	3	15	18	July	11	19	43	July	18	19	59	July	25	11	0
Aug.	2	5	59	Aug.	10	8	50	Aug.	17	3	16	Aug.	23	20	35
Aug.	31	22	2	Sept.	8	19	39	Sept.	15	11	10	Sept.	22	9	43
Sept.	30	14	32	Oct.	8	4	43	Oct.	14	20	22	Oct.	22	2	48
Oct.	30	6	28	Nov.	6	12	47	Nov.	13	7	28	Nov.	20	23	14
Nov.	28	21	15	Dec.	5	20	36	Dec.	12	21	4	Dec.	20	21	9
Dec.	28	10	43												

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TABLE II.- PHASES OF THE MOON - Concluded

[Universal time]

(k) 1971

New moon				First quarter				Full moon				Last quarter			
	d	h	m		d	h	m		d	h	m		d	h	m
				Jan.	4	4	55	Jan.	11	13	21	Jan.	19	18	9
Jan.	26	22	55	Feb.	2	14	31	Feb.	10	7	42	Feb.	18	12	14
Feb.	25	9	49	Mar.	4	2	1	Mar.	12	2	33	Mar.	20	2	30
Mar.	26	19	24	Apr.	2	15	46	Apr.	10	20	10	Apr.	18	12	58
Apr.	25	4	2	May	2	7	34	May	10	11	24	May	17	20	15
May	24	12	32	June	1	0	43	June	9	0	4	June	16	1	25
June	22	21	58	June	30	18	11	July	8	10	37	July	15	5	47
July	22	9	15	July	30	11	7	Aug.	6	19	43	Aug.	13	10	55
Aug.	20	22	54	Aug.	29	2	56	Sept.	5	4	3	Sept.	11	18	23
Sept.	19	14	43	Sept.	27	17	18	Oct.	4	12	20	Oct.	11	5	30
Oct.	19	7	59	Oct.	27	5	55	Nov.	2	21	20	Nov.	9	20	52
Nov.	18	1	46	Nov.	25	16	37	Dec.	2	7	48	Dec.	9	16	3
Dec.	17	19	3												

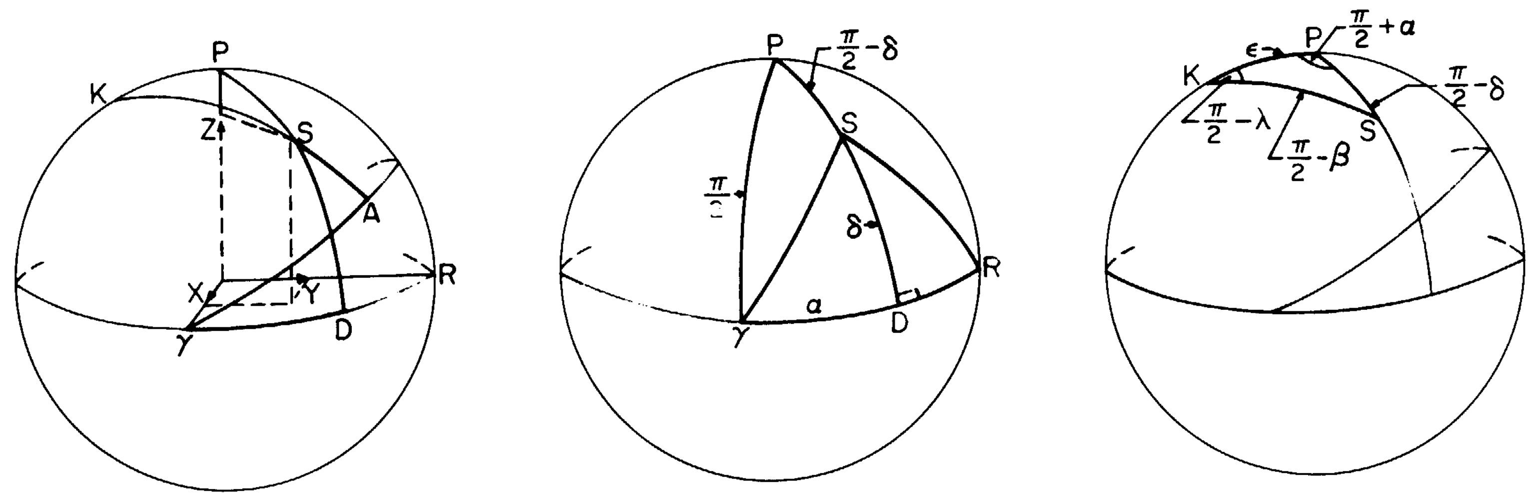

Figure 1.- Rectangular coordinate system and spherical triangles involved in relating equatorial and ecliptic coordinates.

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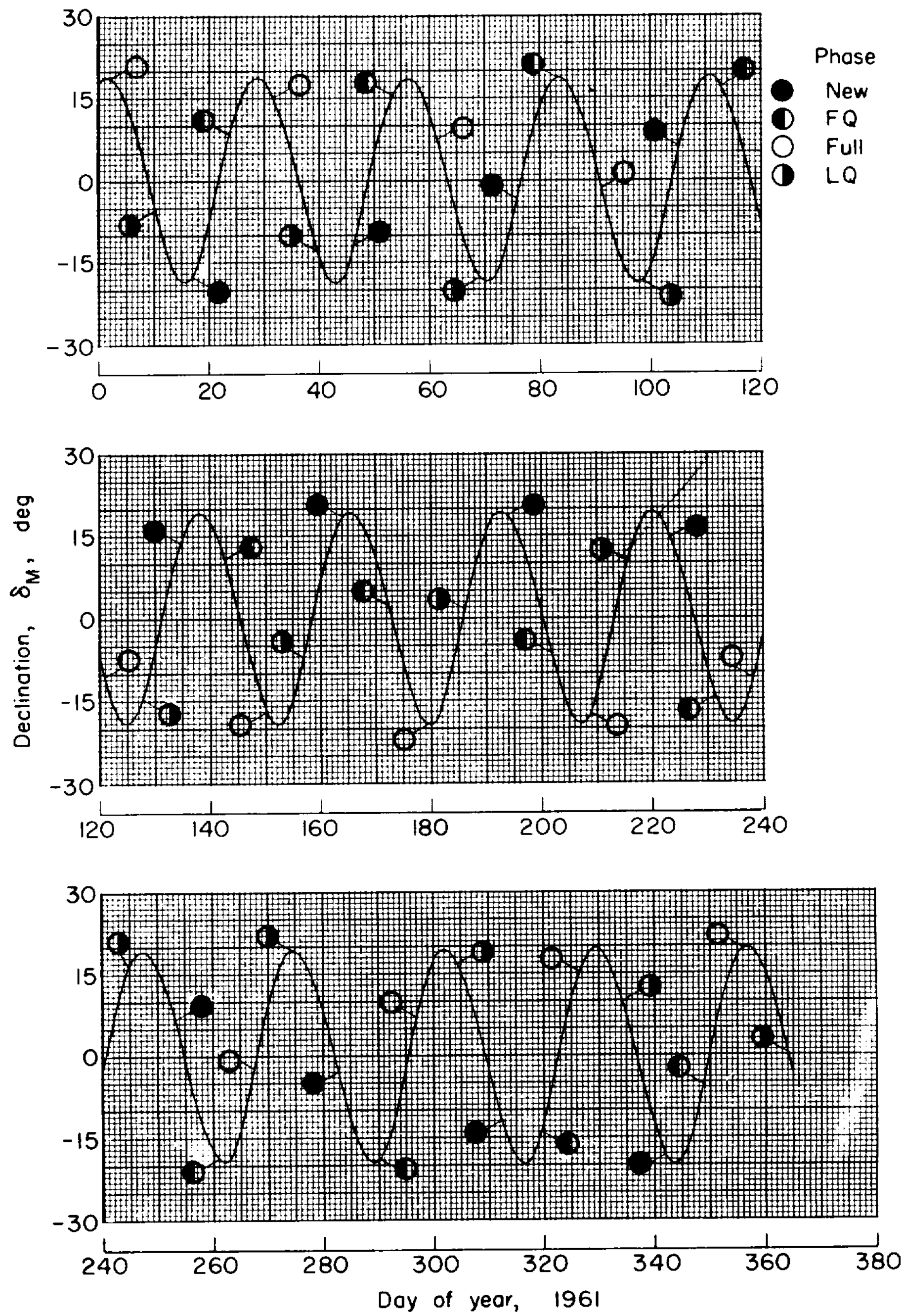
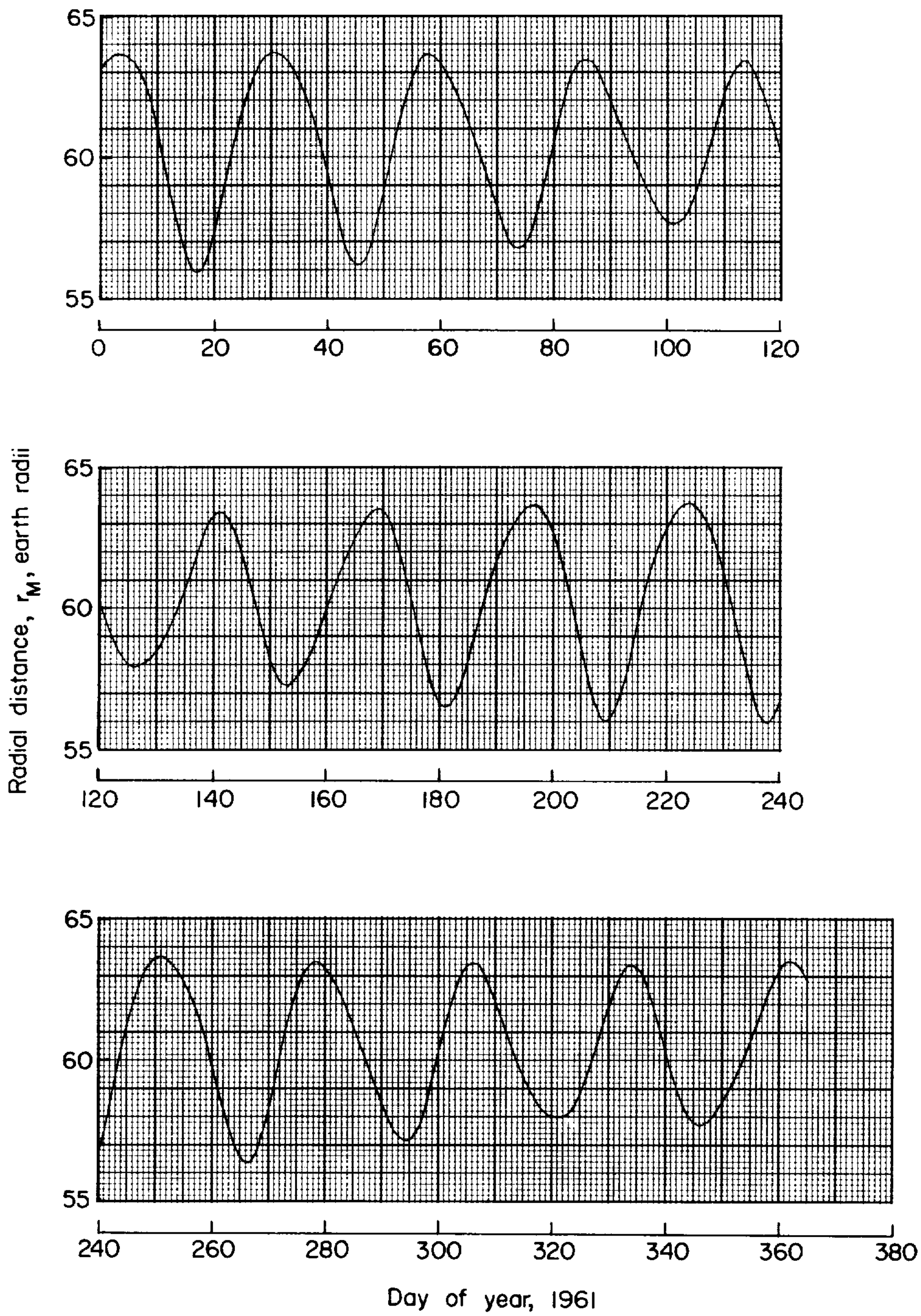


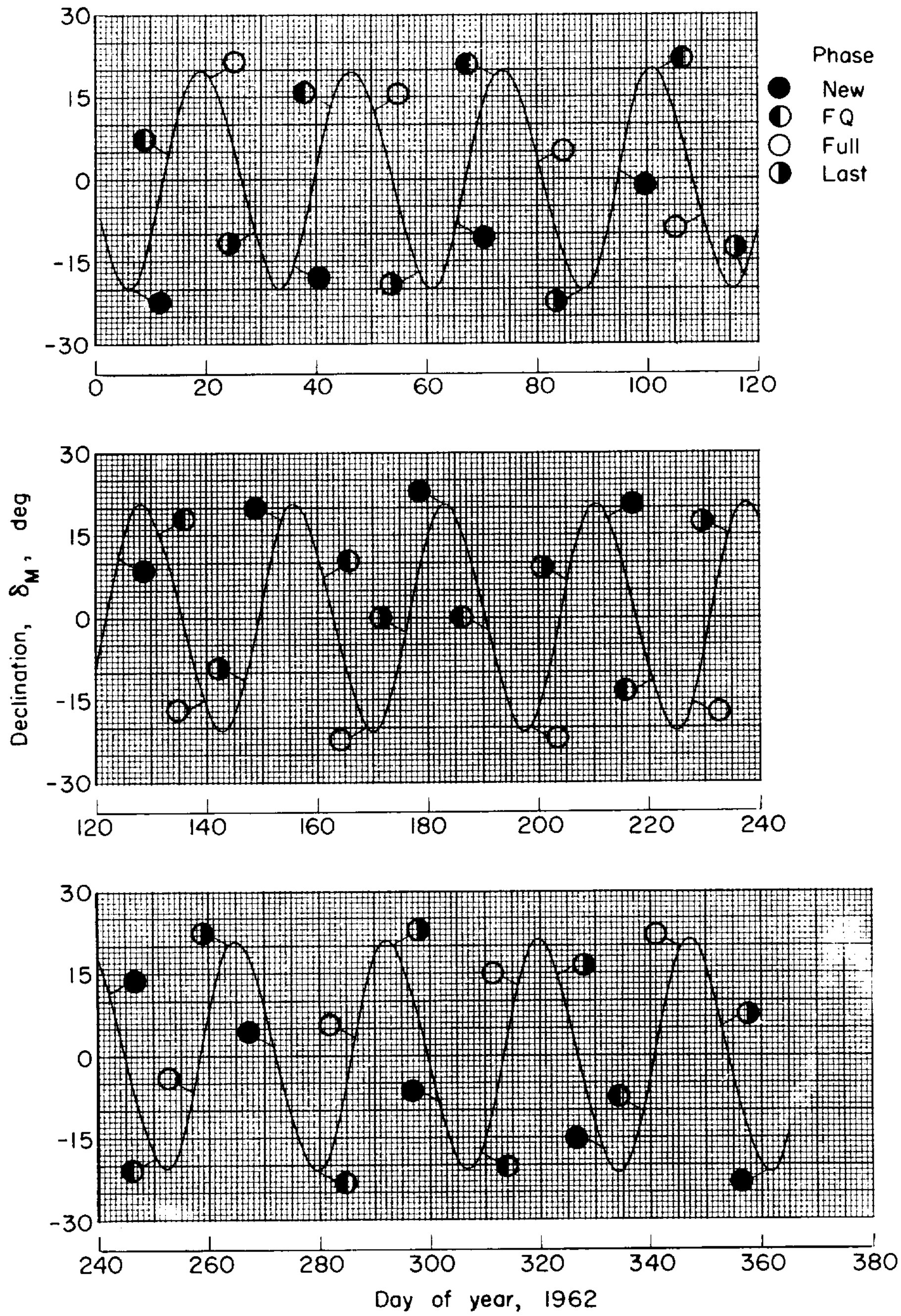
(a) Declination and phases.

Figure 2.- Declination and phases and radial distance of moon for year 1961.



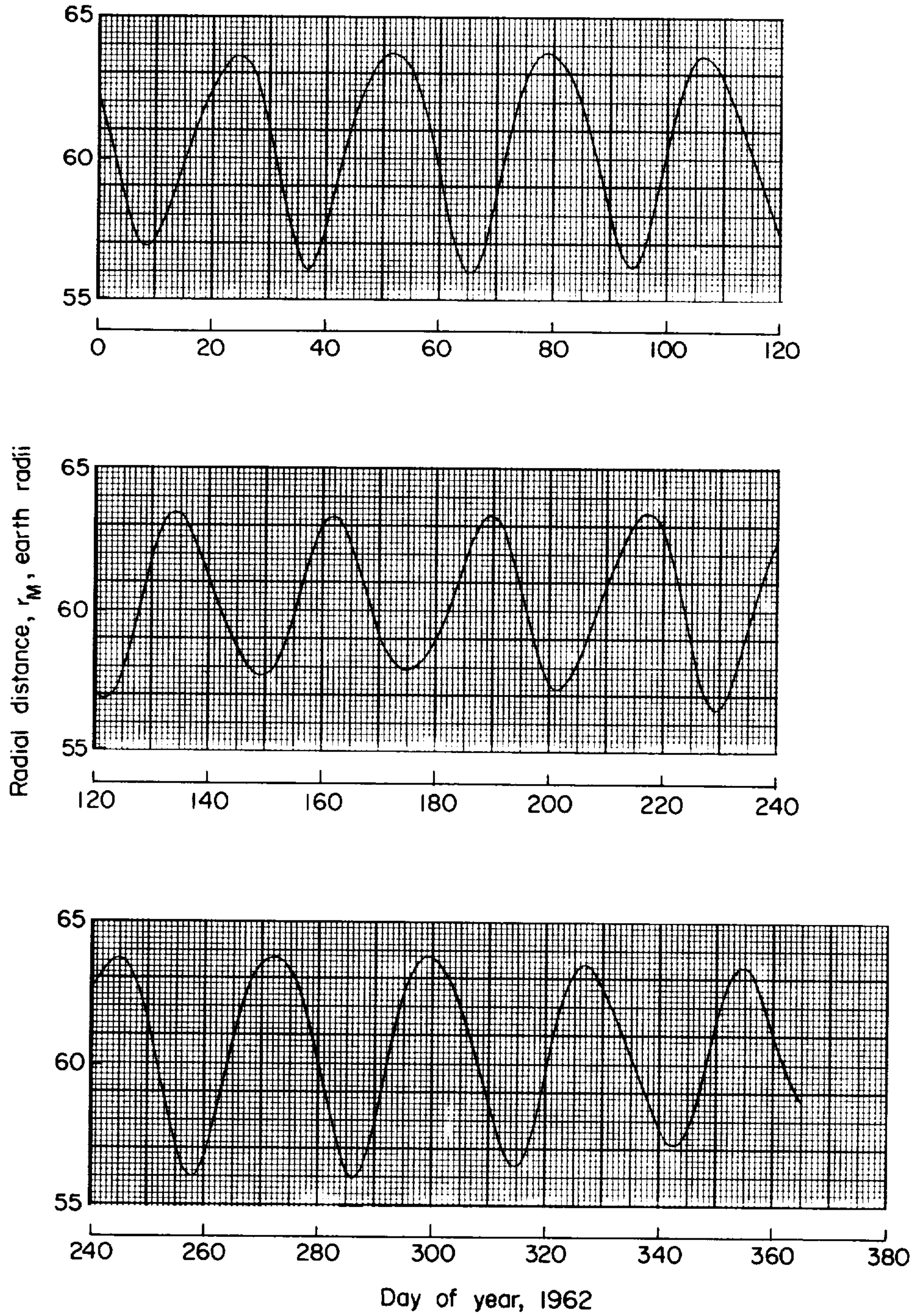
(b) Radial distance.

Figure 2.- Concluded.



(a) Declination and phases.

Figure 3.- Declination and phases and radial distance of moon for year 1962.



(b) Radial distance

Figure 3.- Concluded.

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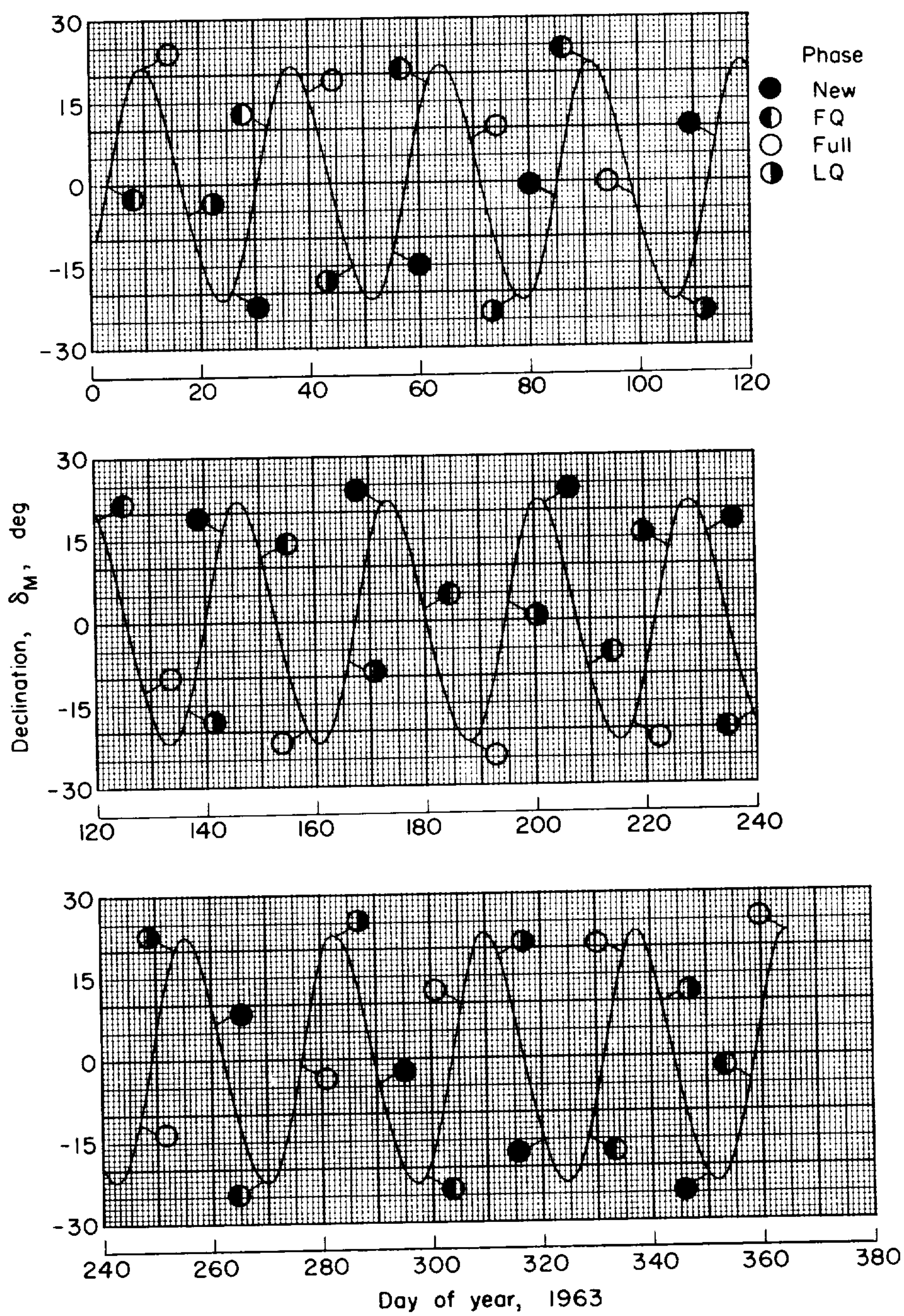
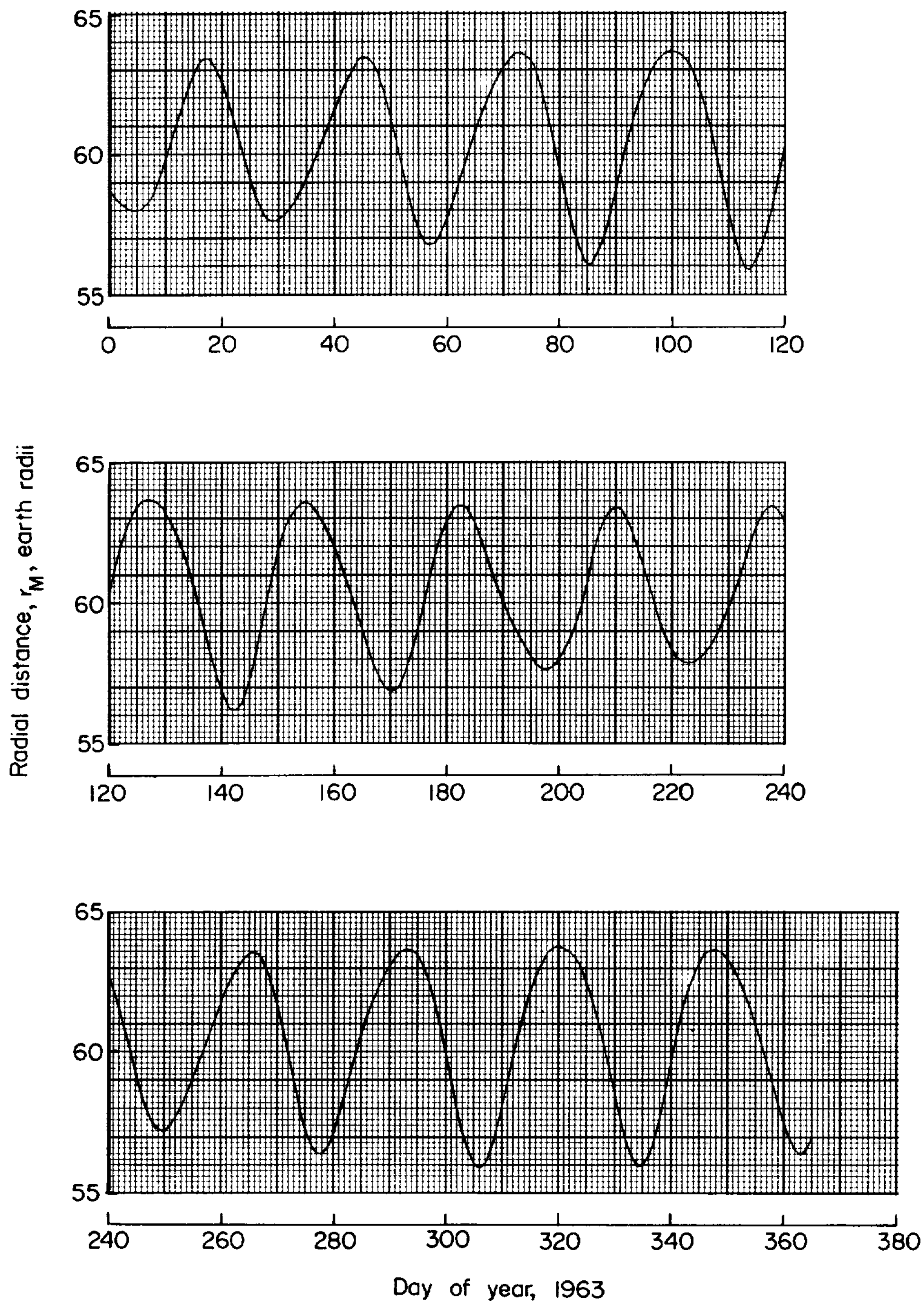


(a) Declination and phases.

Figure 4.- Declination and phases and radial distance of moon for year 1963.



(b) Radial distance.

Figure 4.- Concluded.

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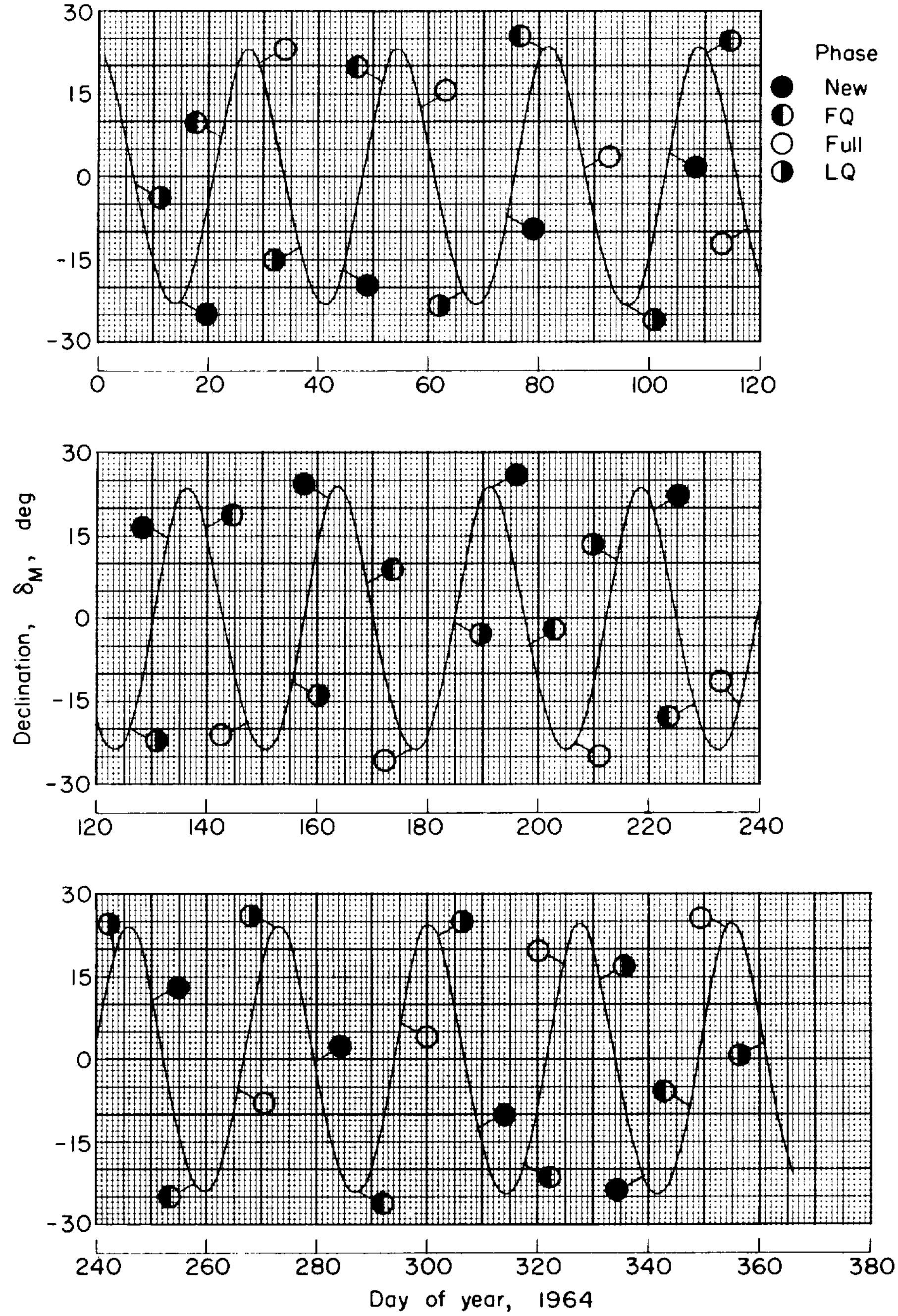
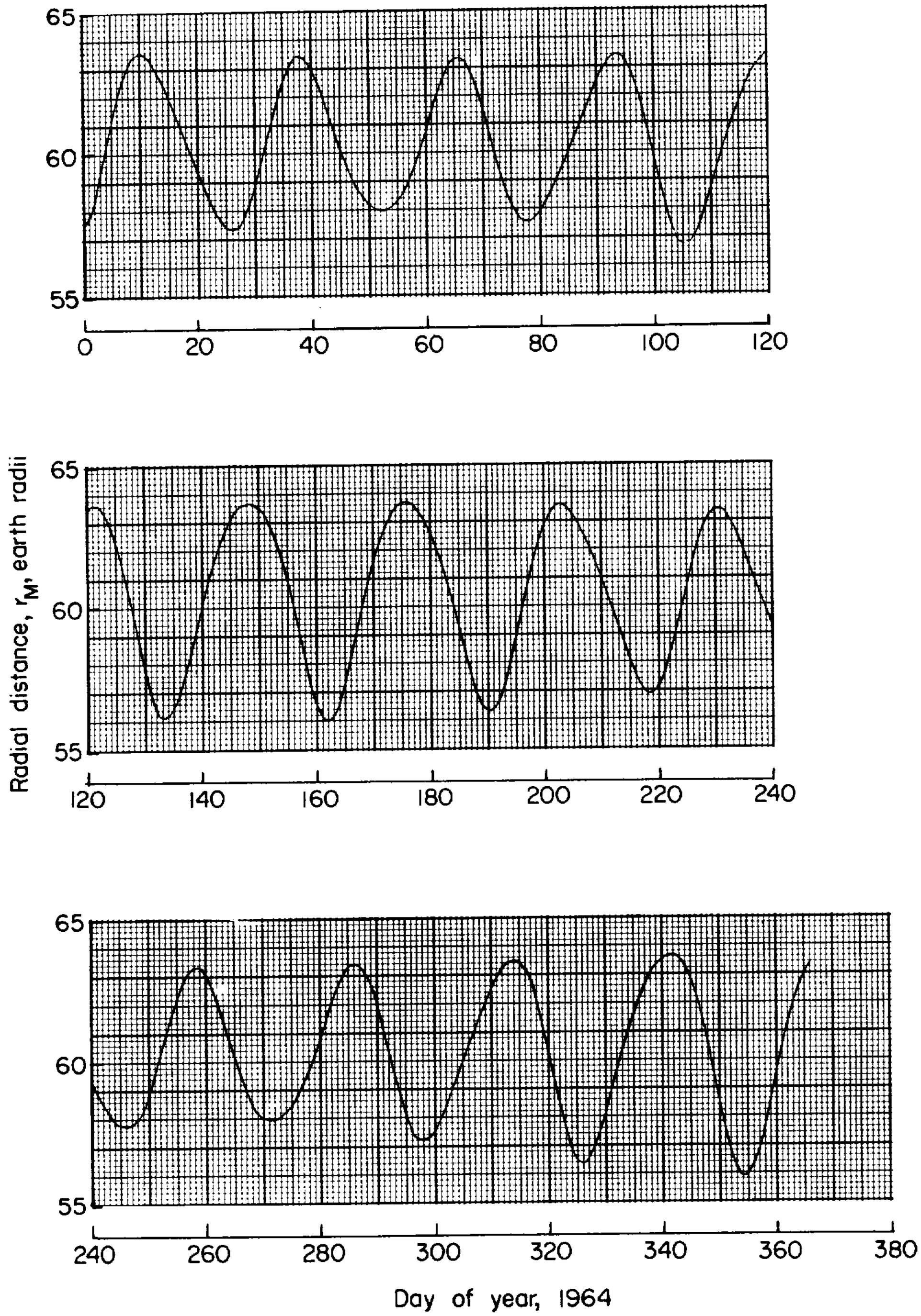


(a) Declination and phases.

Figure 5.- Declination and phases and radial distance of moon for year 1964.



(b) Radial distance.

Figure 5.- Concluded.

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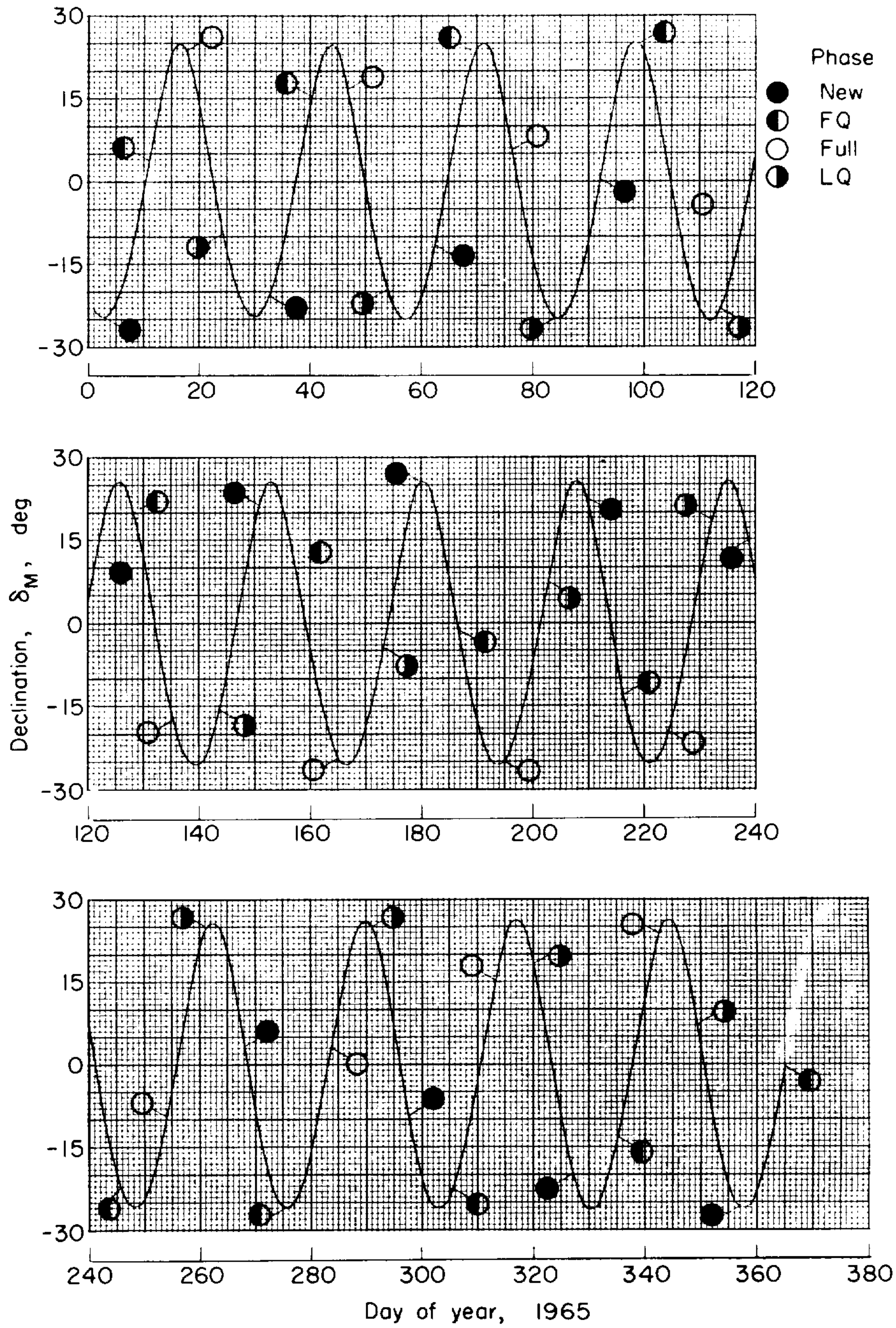
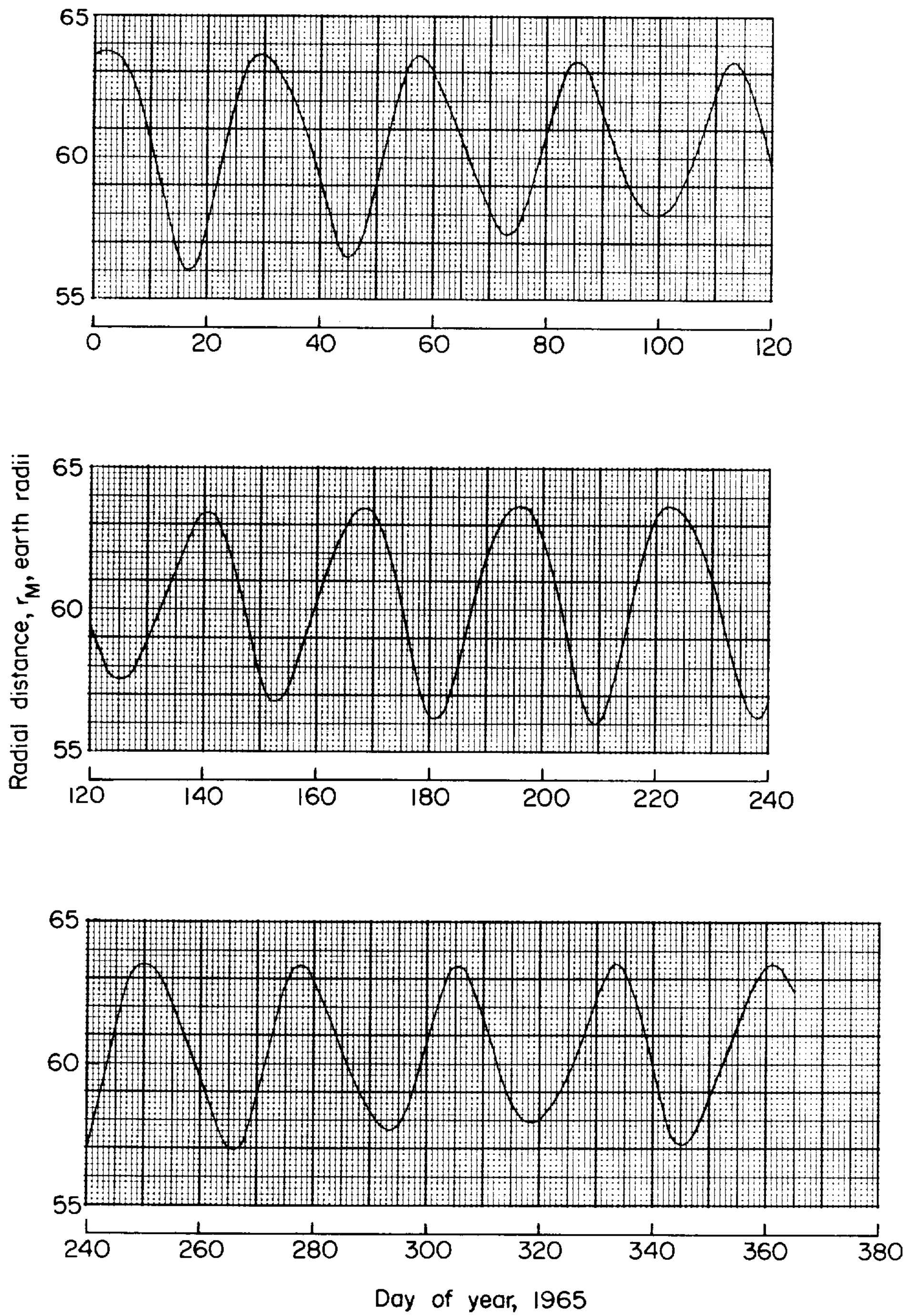


(a) Declination and phases.

Figure 6.- Declination and phases and radial distance of moon for year 1965.



(b) Radial distance.

Figure 6.- Concluded.

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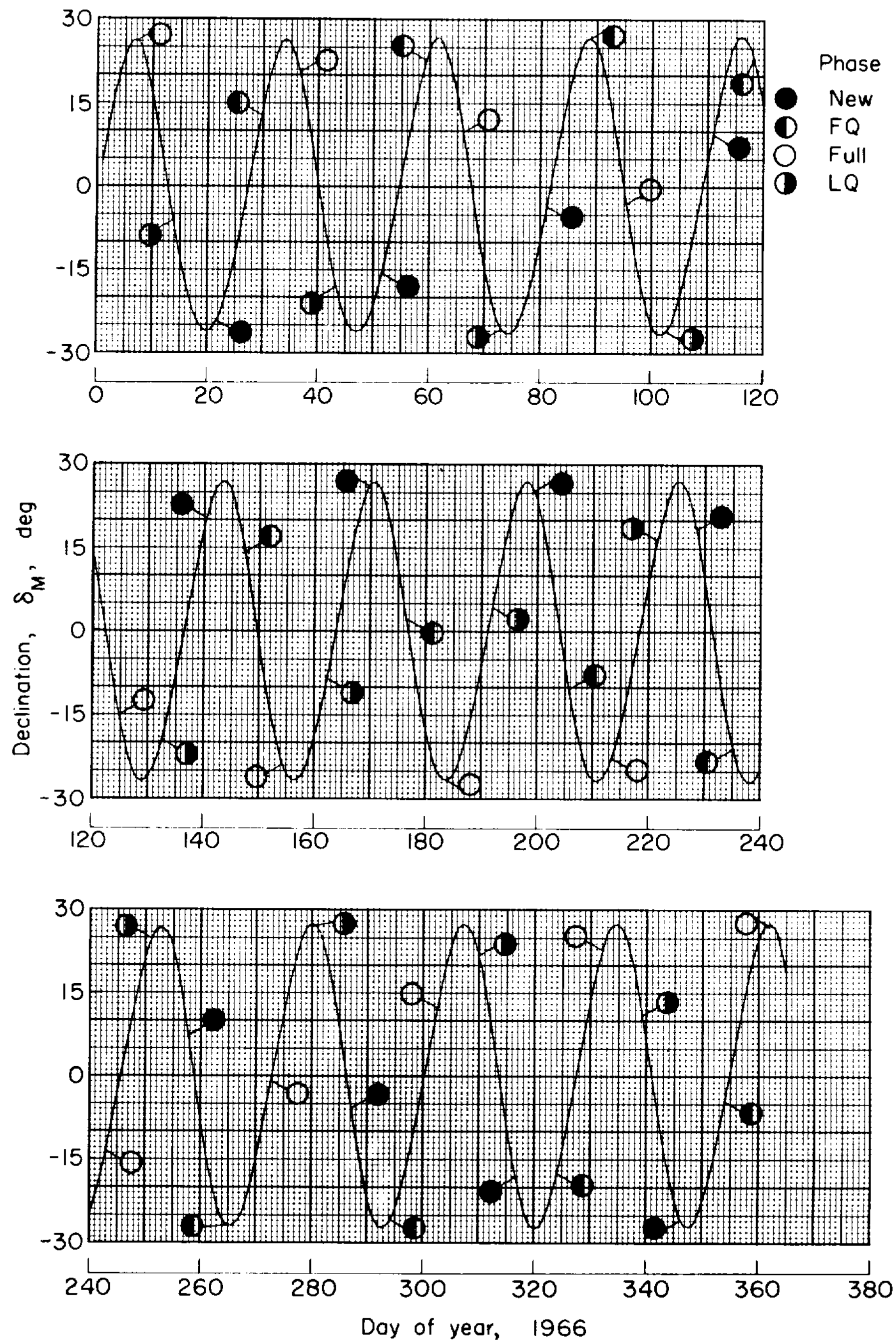
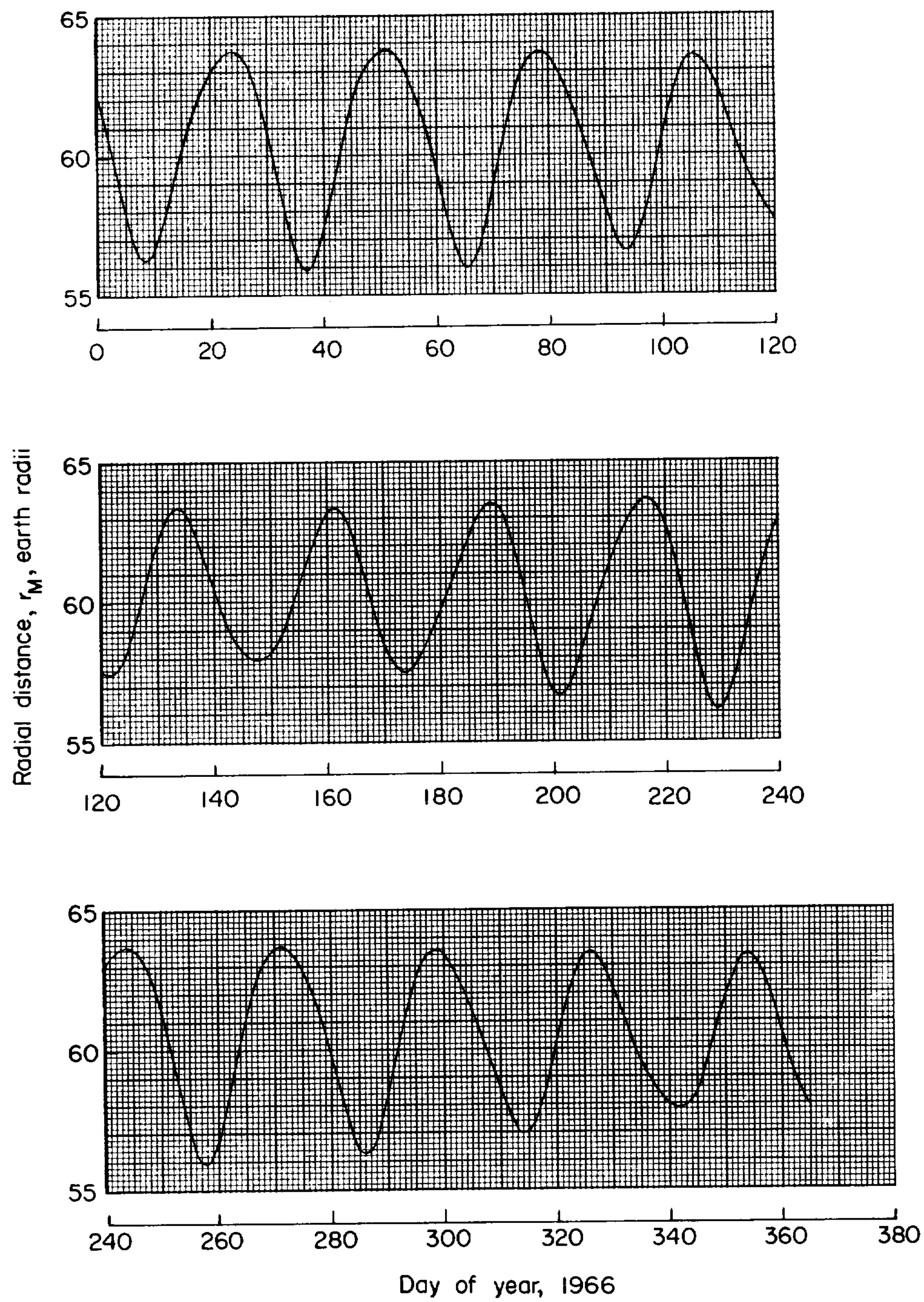


(a) Declination and phases.

Figure 7.- Declination and phases and radial distance of moon for year 1966.



(b) Radial distance.

Figure 7.- Concluded.

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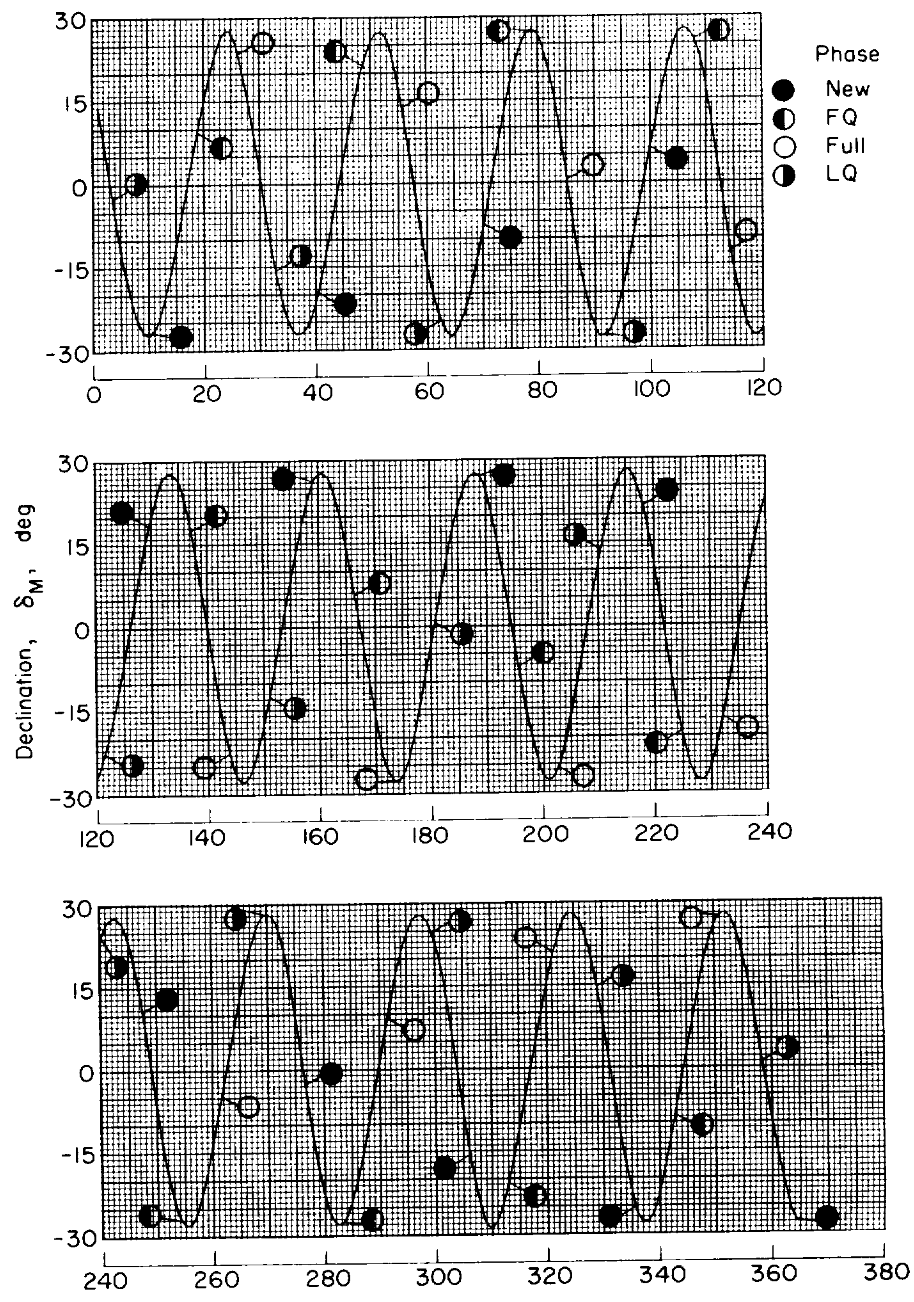
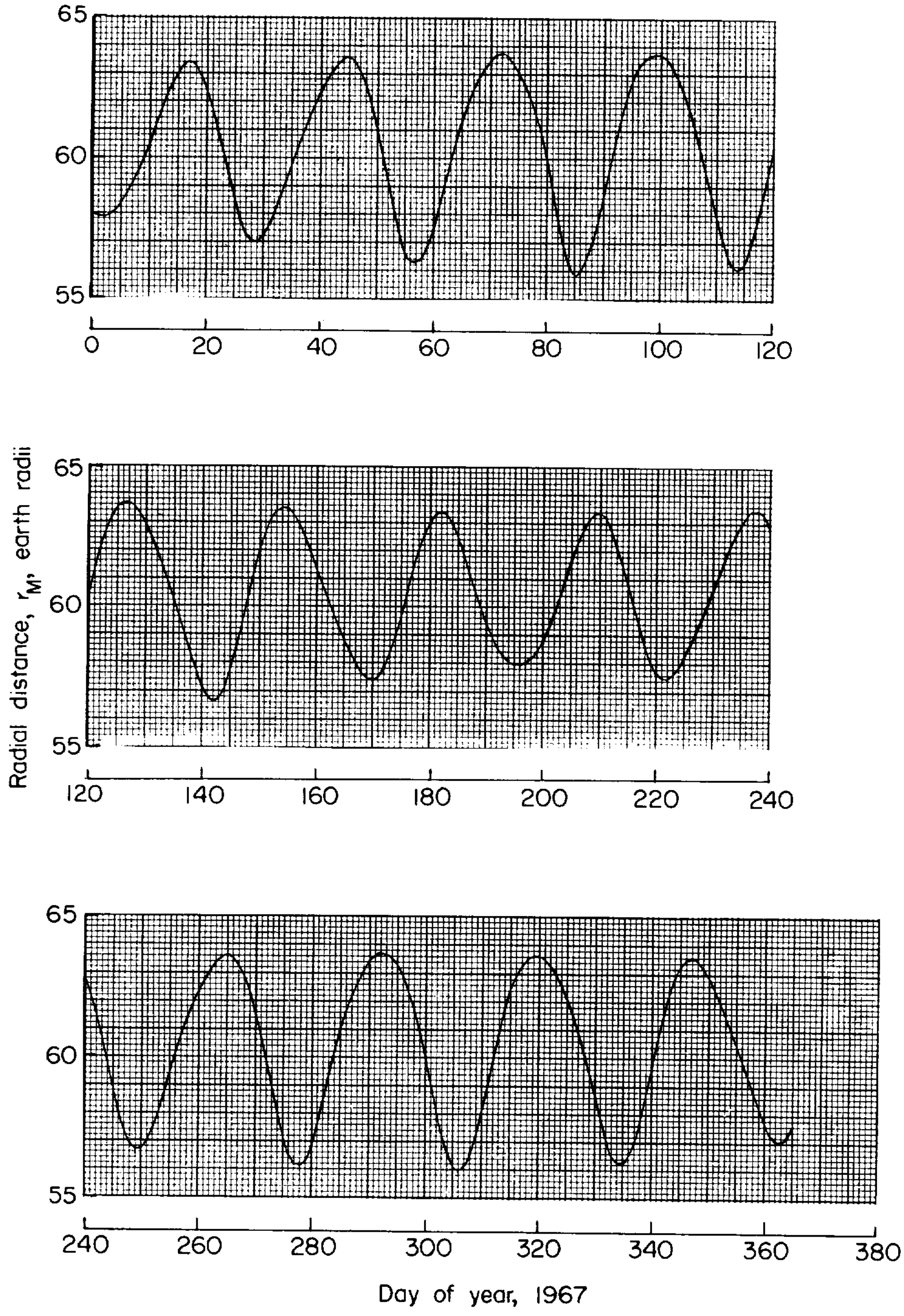


(a) Declination and phases.

Figure 8.- Declination and phases and radial distance of moon for year 1967.



(b) Radial distance.

Figure 8.- Concluded.

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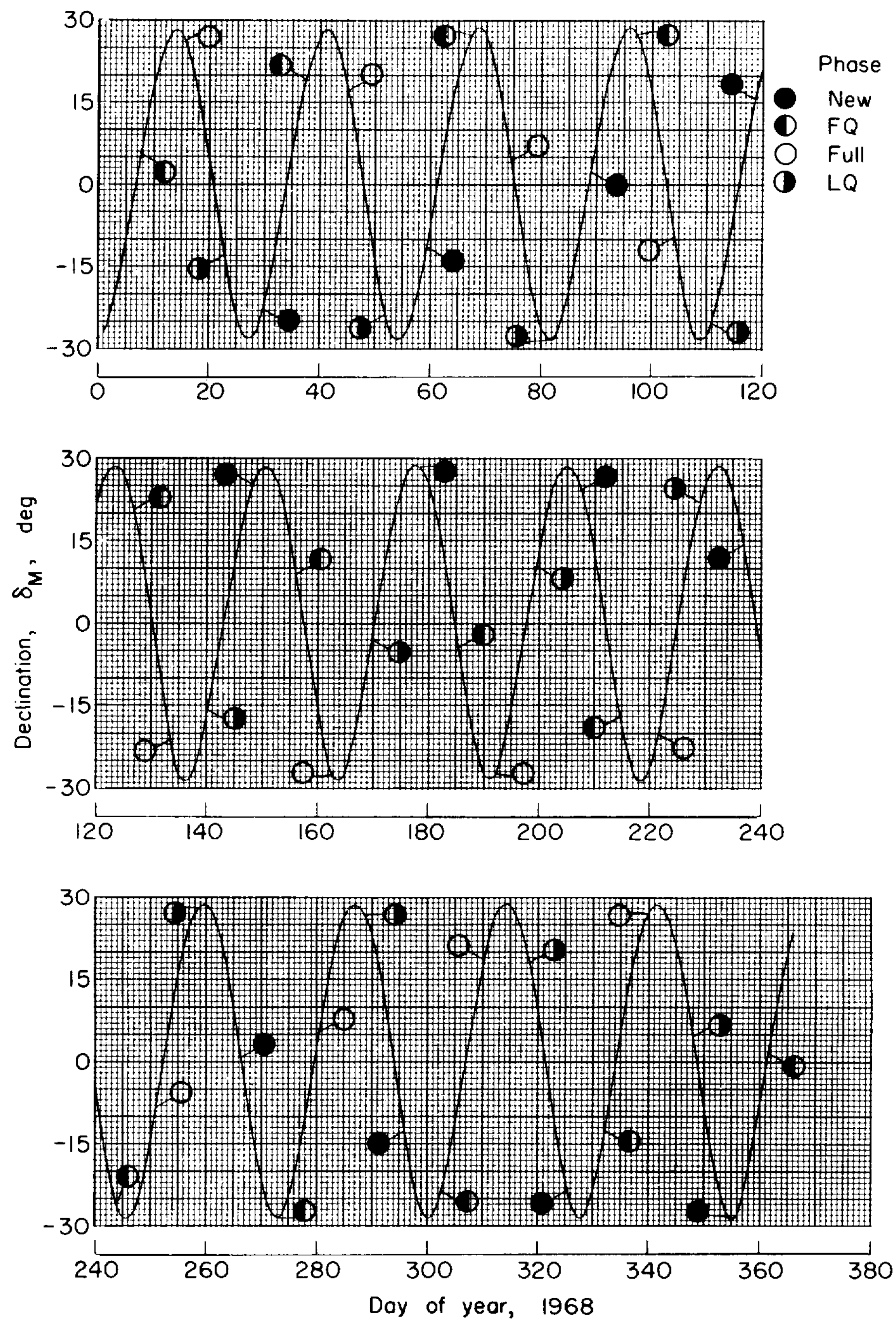
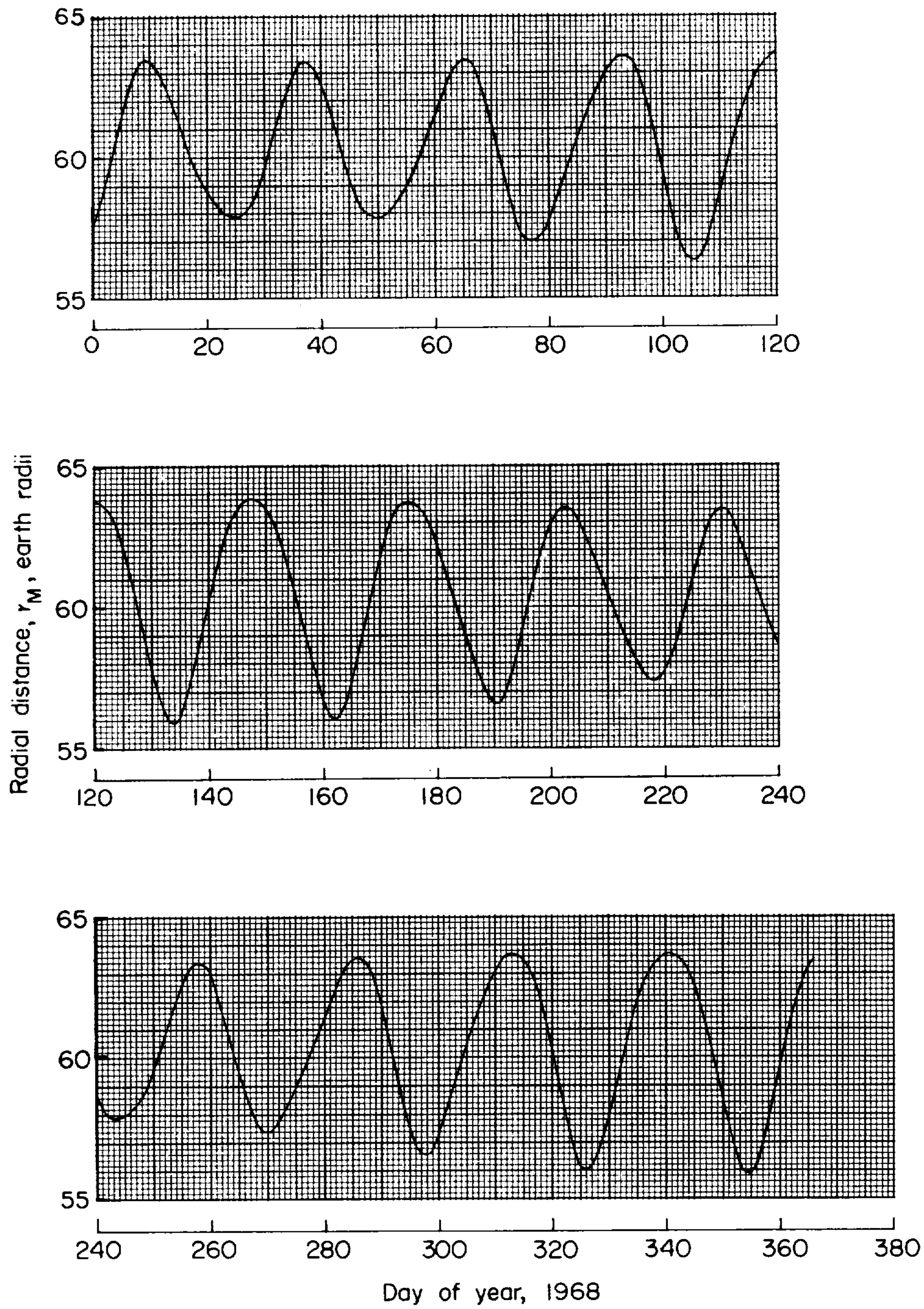


(a) Declination and phases.

Figure 9.- Declination and phases and radial distance of moon for year 1968.

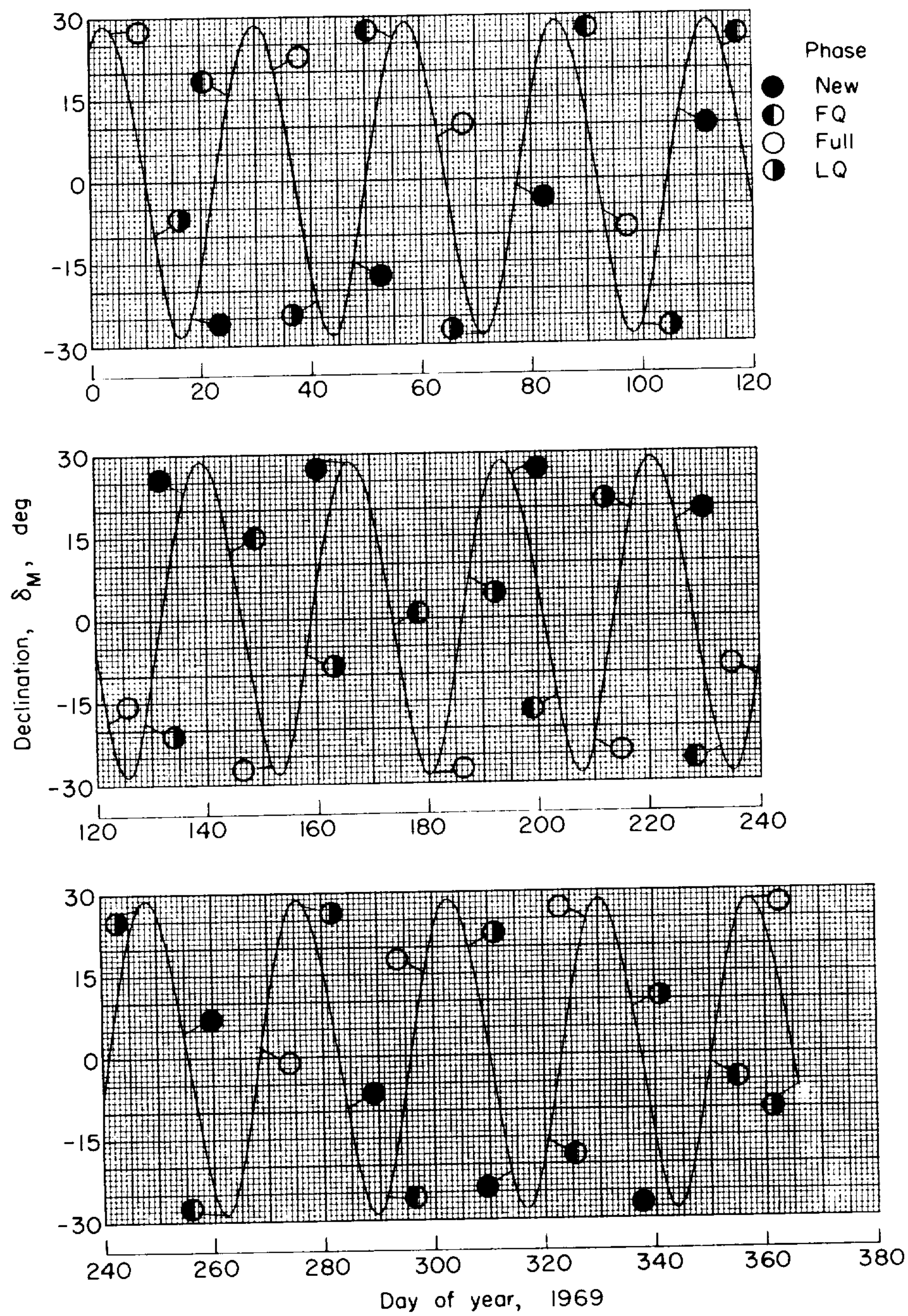


(b) Radial distance.

Figure 9.- Concluded.

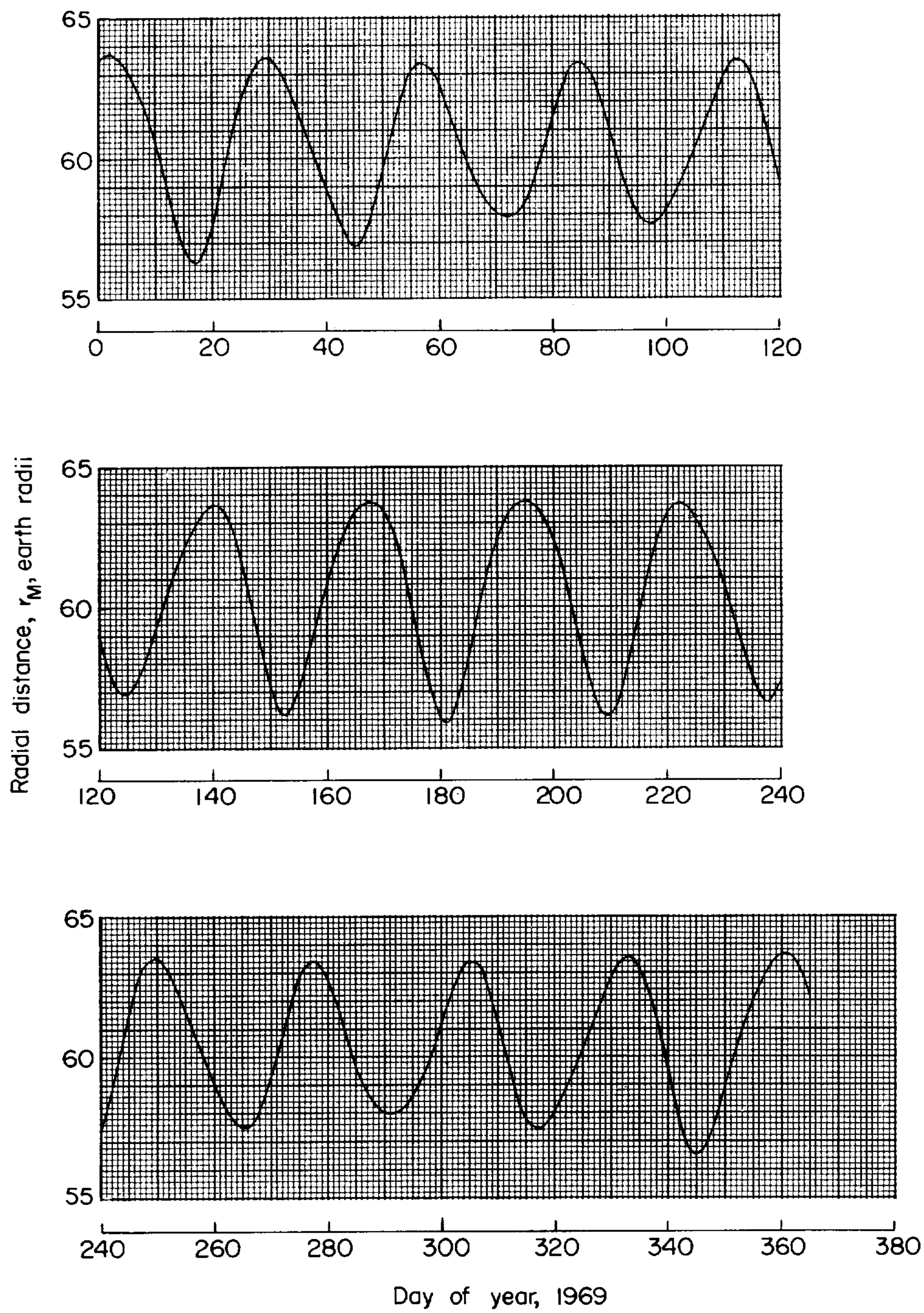
L-1576

L-1576



(a) Declination and phases.

Figure 10.- Declination and phases and radial distance of moon for year 1969.



(b) Radial distance.

Figure 10.- Concluded.

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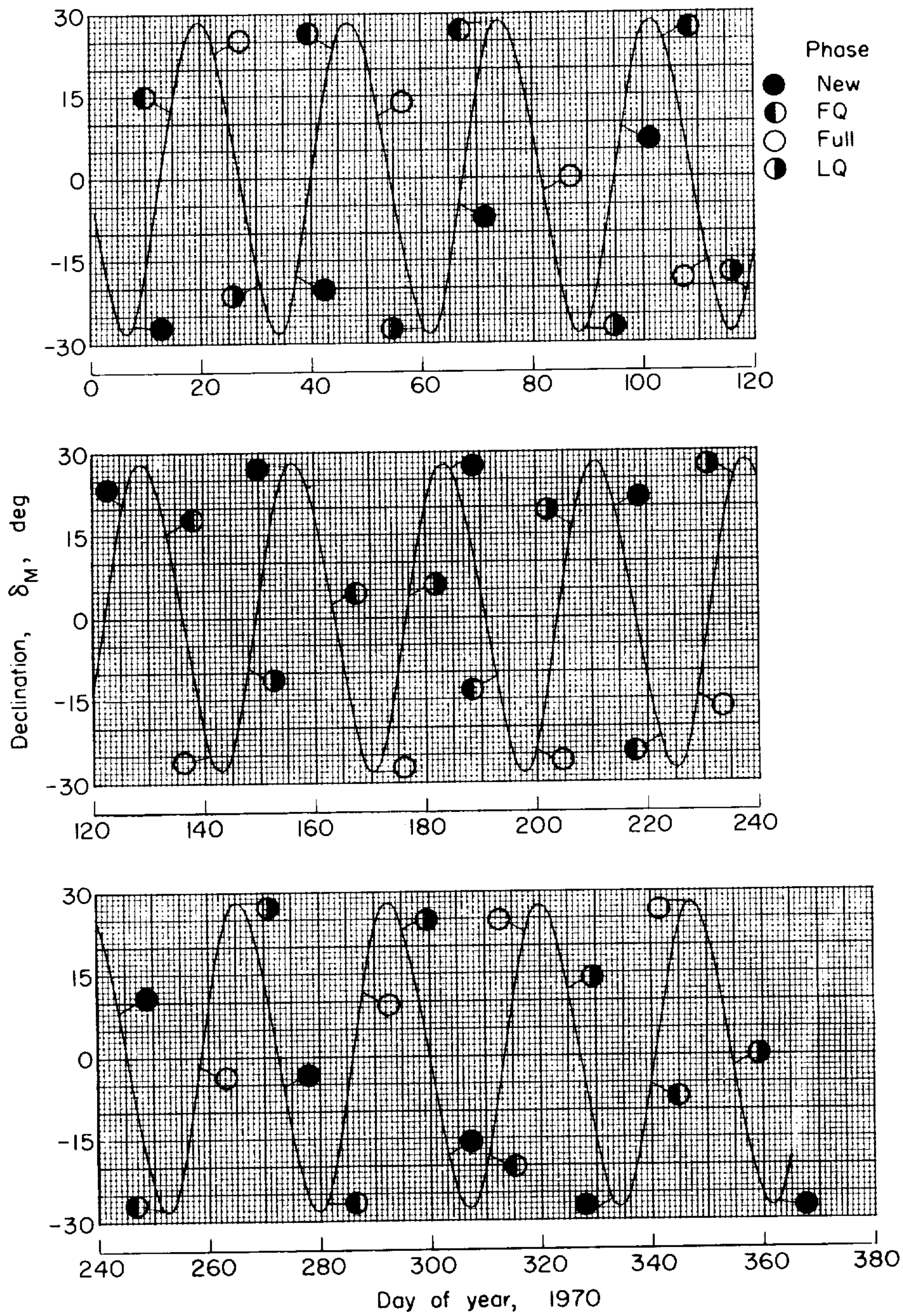
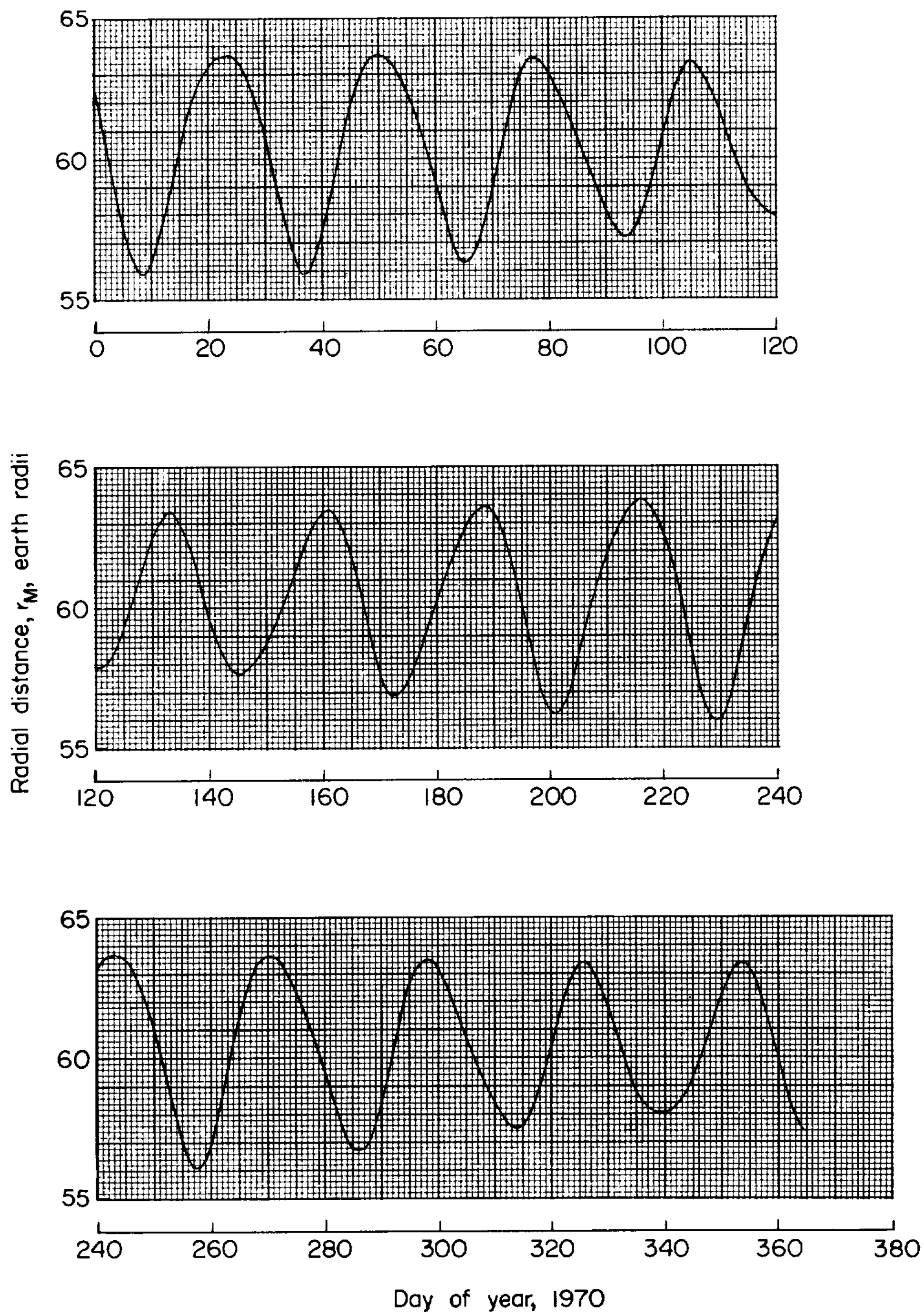


(a) Declination and phases.

Figure 11.- Declination and phases and radial distance of moon for year 1970.



(b) Radial distance.

Figure 11.- Concluded.

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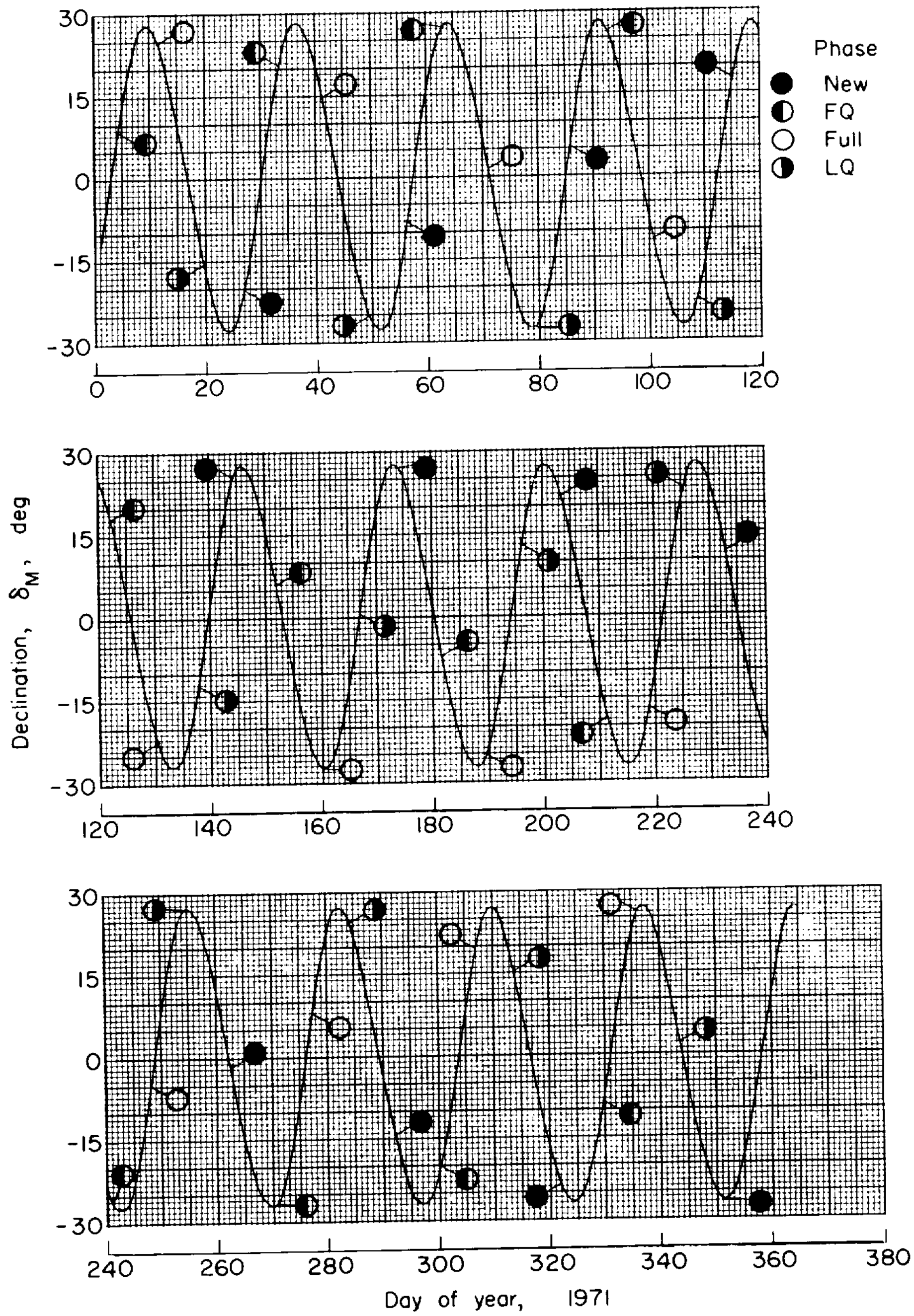
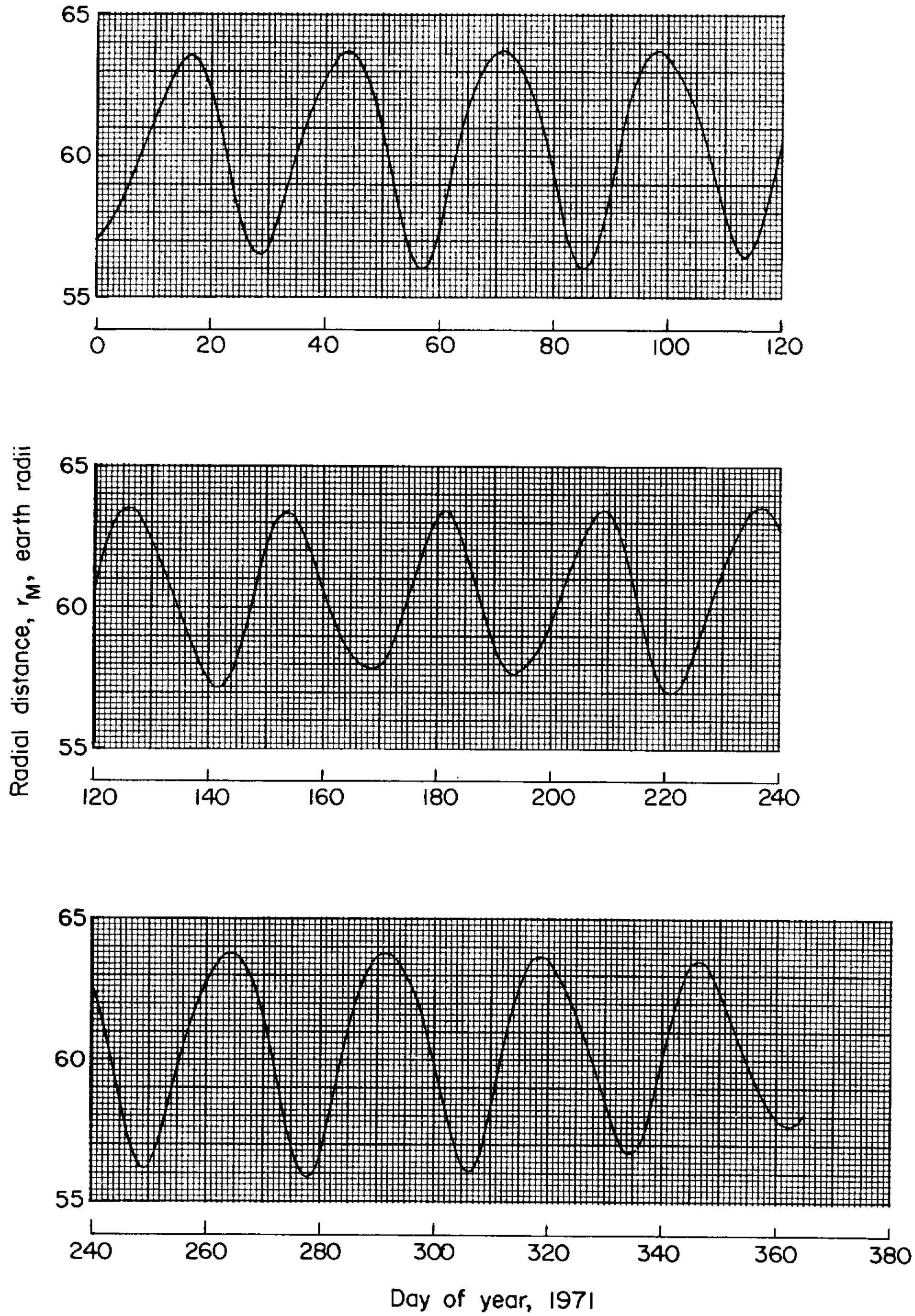


(a) Declination and phases.

Figure 12.- Declination and phases and radial distance of moon for year 1971.



(b) Radial distance.

Figure 12.- Concluded.

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NASA TN D-911
National Aeronautics and Space Administration.
DECLINATION, RADIAL DISTANCE, AND PHASES OF THE MOON FOR THE YEARS 1961 TO 1971 FOR USE IN TRAJECTORY CONSIDERATIONS. Donald S. Woolston. August 1961. 48p. OTS price, $1.25. (NASA TECHNICAL NOTE D-911)

As a byproduct of the preparation of solar and lunar coordinates for use in trajectory calculations a time history has been obtained of the radial distance and declination of the moon and its phases. Results are intended for use as an aid in the selection of launch dates. Results are presented for the years 1961 to 1971 in a form which permits a rapid approximate determination of the combination of declination and lighting for any calendar date. The information provides a time basis for entering tables of the moon's coordinates to obtain more precise data for use in computing insertion conditions.

I. Woolston, Donald S.
II. NASA TN D-911

(Initial NASA distribution: 6, Astronomy; 46, Space mechanics.)

NASA

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NASA TN D-911
National Aeronautics and Space Administration.
DECLINATION, RADIAL DISTANCE, AND PHASES OF THE MOON FOR THE YEARS 1961 TO 1971 FOR USE IN TRAJECTORY CONSIDERATIONS. Donald S. Woolston. August 1961. 48p. OTS price, $1.25. (NASA TECHNICAL NOTE D-911)

As a byproduct of the preparation of solar and lunar coordinates for use in trajectory calculations a time history has been obtained of the radial distance and declination of the moon and its phases. Results are intended for use as an aid in the selection of launch dates. Results are presented for the years 1961 to 1971 in a form which permits a rapid approximate determination of the combination of declination and lighting for any calendar date. The information provides a time basis for entering tables of the moon's coordinates to obtain more precise data for use in computing insertion conditions.

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